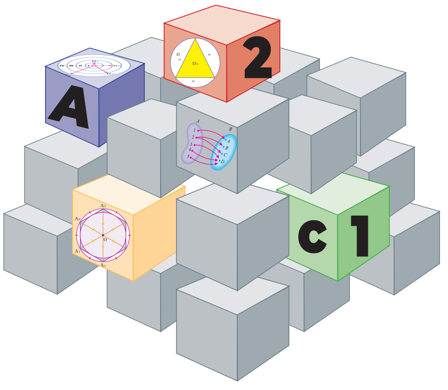
ALGEBRA
VA ANALIZ ASOSLARI
10



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MUNDARIJA

10-SINF "ALGEBRA VA ANALIZ ASOSLARI" DARSLIGI UCHUN TA'LIMIY ILOVA

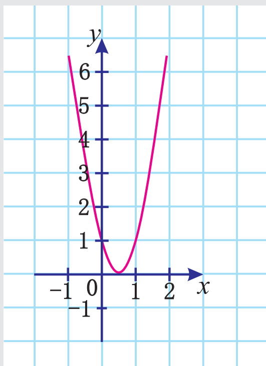
10-SINF "ALGEBRA VA ANALIZ ASOSLARI" DARSLIGI UCHUN VIDEODARSLAR

TAKRORLASH

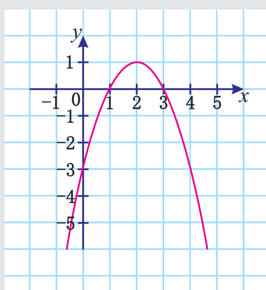
- KVADRAT FUNKSIYA
- KVADRAT TENGSIZLIK
- TRIGONOMETRIK AYNIYATLAR
- ARIFMETIK VA GEOMETRIK PROGRESSIYALAR

KVADRAT FUNKSIYA

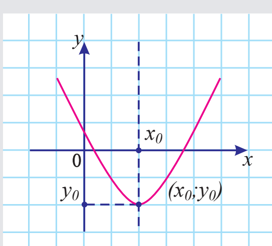
1-rasm



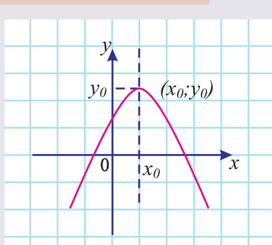
2-rasm



3-rasm



4-rasm



Kvadrat funksiyaning ta'rifi

Ta'rif

$y = ax^2 + bx + c$ ko'rinishidagi funksiya ***kvadrat funksiya*** deyiladi, bunda *a, b, c* berilgan haqiqiy sonlar, $a \neq 0$, x – haqiqiy o'zgaruvchi.

Masalan, quyidagi funksiyalar kvadrat funksiyalardir:
$y = 3x^2 + 2x - 1$, $y = -4x^2 - 5x$, $y = 6x^2 - 3$, $y = 4x^2$, $y = 2 - x^2$.

Kvadrat funksiyaning grafigi

1. $y = ax^2 + bx + c$ kvadrat funksiyaning grafigi *parabola* deb ataladigan egri chiziqdan iborat bo'ladi. 1-rasmda $y = 4x^2 - 4x + 1$ va 2-rasmda $y = -x^2 + 4x - 3$ funksiyalar grafiklari tasvirlangan.
2. $y = ax^2 + bx + c$ parabola tarmoqlari $a > 0$ bo'lganda (3-*rasm*) ordinata o'qi bo'yicha yuqoriga yo'nalgan, $a < 0$ bo'lganda (4-*rasm*) esa pastga yo'nalgan bo'ladi.
3. $y = ax^2 + bx + c$ parabola uchining $(x_0; y_0)$ koordinatalari $x_0 = -\dfrac{b}{2a}$, $y_0 = ax_0^2 + bx_0 + c$ yoki $y_0 = -\dfrac{b^2 - 4ac}{4a}$ formulalar bilan hisoblanadi.
4. $y = ax^2 + bx + c$ parabola o'zining uchi orqali o'tuvchi va ordinata o'qiga parallel to'g'ri chiziqqa nisbatan simmetrik bo'ladi.
5. $y = ax^2 + bx + c$ parabolaning Ox o'qi bilan kesishish nuqtalarining abssissalari kvadrat funksiyaning nollari bo'ladi. Kvadrat funksiya nollarini topish uchun $ax^2 + bx + c = 0$ tenglamani yechish kerak.
6. Kvadrat funksiyaning Oy o'qi bilan kesishish nuqtasining ordinatasi funksiyaning $x = 0$ nuqtadagi qiymatidan iborat.

$y = ax^2 + bx + c$ kvadrat funksiyaning grafigini yasash uchun:

1. Parabola tarmoqlari yo'nalishi aniqlanadi.
2. Parabola uchining koordinatalari $x_0 = -\dfrac{b}{2a}$, $y_0 = ax_0^2 + bx_0 + c$ formulalar yordamida topiladi va koordinata tekisligida belgilanadi.
3. Parabolaning abssissa o'qi bilan kesishish nuqtalari (nollari) topiladi. Agar funksiya nollari mavjud bo'lmasa, u holda odatda parabolaning simmetriya o'qiga nisbatan simmetrik bo'lgan ikkita nuqta topiladi. Masalan: parabolaning Oy o'qi bilan kesishish nuqtasi (0; c) va unga simmetrik bo'lgan ($2x_0$; c).
4. Yasalgan nuqtalarni uzluksiz silliq egri chiziq bilan tutashtiriladi (parabolaning grafigini aniqroq yasash uchun yana bir nechta nuqtasini yasash maqsadga muvofiq bo'ladi).

Kvadrat funksiyaning xossalari

1. Aniqlanish sohasi:

$D(y)=(-\infty;\infty)$.

2. Qiymatlar to‘plami:

a) $a>0$ bo‘lsa, $E(y)=[y_0;\infty)$.

b) $a<0$ bo‘lsa, $E(y)=(-\infty;y_0]$.

3. Eng katta va eng kichik qiymatlari:

a) $a>0$ bo‘lsa, $x=x_0$ nuqtada eng kichik qiymatga erishadi va bu qiymat $y_0=ax_0^2+bx_0+c$ ga teng bo‘ladi, eng katta qiymatga esa erishmaydi.

b) $a<0$ bo‘lsa, $x=x_0$ nuqtada eng katta qiymatga erishadi va bu qiymat $y_0=ax_0^2+bx_0+c$ ga teng bo‘ladi, eng kichik qiymatga esa erishmaydi.

4. Funksiya nollari:

a) $D=b^2-4ac>0$ bo‘lsa, funksiya ikkita nollarga ega: $x_1=\dfrac{-b+\sqrt{D}}{2a}$ va $x_2=\dfrac{-b-\sqrt{D}}{2a}$.

b) $D=b^2-4ac=0$ bo‘lsa, funksiya bitta (o‘zaro teng ikkita) nolga ega: $x=\dfrac{-b}{2a}$.

c) $D=b^2-4ac<0$ bo‘lsa, funksiya nollarga ega emas.

5. Monotonlik oraliqlari:

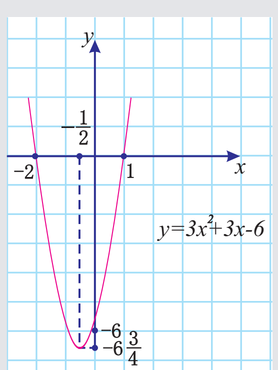
a) $a>0$ bo‘lsa, $y=ax^2+bx+c$ funksiya $(-\infty;x_0]$ da kamayuvchi, $[x_0;\infty)$ da o‘suvchi bo‘ladi.

b) $a<0$ da $y=ax^2+bx+c$ funksiya $(-\infty;x_0]$ da o‘suvchi, $[x_0;\infty)$ da kamayuvchi bo‘ladi (bu yerda x_0 – parabola uchining abssissasi).

1-misol. $y=3x^2+3x-6$ kvadrat funksiya berilgan bo‘lsin. Uning xossalarini yozing va grafigini yasab ko‘rsating.

Yechish

5-rasm



1. Aniqlanish sohasi: $D(y)=(-\infty;\infty)$.
2. $a=3>0$ va $x_0=-\dfrac{1}{2}$, $y_0=-6,75$, $E(y)=[-6,75;\infty)$.
3. $x=-\dfrac{1}{2}$ bo‘lganda eng kichik qiymati $y=-6,75$ ga teng, eng katta qiymatga erishmaydi.
4. $D=81>0$, demak, nollari ikkita: $x_1=1, x_2=-2$.
5. $x\in(-\infty;-2)\cup(1;\infty)$ da $y>0$ va $x\in(-2;1)$ da $y<0$ bo‘ladi.
6. Funksiya juft ham, toq ham emas.
7. Funksiya $\left(-\infty;-\dfrac{1}{2}\right]$ oraliqda kamayuvchi, $\left[-\dfrac{1}{2};\infty\right)$ oraliqda o‘suvchi bo‘ladi.

Funksiya grafigi 5-rasmda ko‘rsatilgan.

MISOLLAR

1. Qaysi funksiyalar kvadrat funksiya bo'ladi?

a) $y=\frac{1}{3}x+2$ b) $y=-x^2+5x+1$ c) $y=x^2-x^3$ d) $y=x^2$

2. $x=-3$ bo'lganda, $y=4x^2+7x-5$ funksiyaning qiymati nechaga teng bo'ladi?

3. x ning qanday qiymatlarida $y=-3x^2+x+1$ funksiyaning qiymati –1 ga teng bo'ladi?

4. $y=-5x^2+x+\sqrt{7}$ funksiya x ning qanday qiymatlarida aniqlangan?

5. –5 soni $y=x^2-5x$ funksiyaning noli bo'ladimi?

6. Funksiya grafigini yasang.

a) $y=x^2$ b) $y=-x^2$ c) $y=3x^2$
d) $y=-3x^2-5$ e) $y=x^2-2x$ f) $y=-2x^2+5x$

7. Funksiya nollarini toping.

a) $y=2x^2+5x+2$ b) $y=3x^2+10x+3$ c) $y=-2x^2+x-5$

8. Funksiyaning qiymatlar to'plamini toping.

a) $y=x^2+2$ b) $y=(x-4)^2-1$ c) $y=(x-5)^2+3$ d) $y=3-4x^2$
e) $y=3x-x^2$ f) $y=3x^2+2x$ g) $y=2x^2-8x+19$ h) $y=-3x^2-12x+1$

9. x ning qanday qiymatlarida funksiya eng katta (yoki eng kichik) qiymat qabul qilishini aniqlang va uni toping.

a) $y=x^2+9x+34$ b) $y=-9x^2-3x+7$ c) $y=-2x^2-5x+1$

10. t ning qanday qiymatlarida $y=2x^2-tx+8$ funksiyaning nollari mavjud emas?

11. x ning qanday qiymatlarida $y=5x^2-4x-1$ funksiyaning qiymatlari manfiy bo'ladi?

12. $y=x^2+6x+13$ funksiya manfiy qiymatlarni qabul qila oladimi?

13. $y=-x^2-4x-5$ funksiya musbat qiymatlarni qabul qila oladimi?

14. $y=6x^2+7x+1$ funksiya grafigini yasang va grafik bo'yicha funksiyaning qiymatlari musbat, manfiy bo'ladigan x ning qiymatlarini toping.

15. $y=-x^2+4x-3$ funksiya grafigini yasang. Grafik yordamida funksiyaning o'sish va kamayish oraliqlarini toping.

16. x ning qanday qiymatlarida $y=x^2-22x+27$ va $y=2x^2-20x+3$ funksiyalarning qiymatlari teng bo'ladi?

17. Agar parabolaning $(-1;\ 6)$ nuqta orqali o'tishi va uning uchi $(1;\ 2)$ nuqtada ekani ma'lum bo'lsa, parabolaning tenglamasini toping.

18. $y=x^2+px+q$ parabolaning uchi $A(1;\ -2)$ bo'lsa, p va q koeffitsiyentlarni toping.

19. Agar $y=ax^2+bx+c$ parabolaning uchi $M(-1;\ -7)$ va parabola ordinatalar o'qi bilan $N(0;\ -4)$ nuqtada kesishsa, $a,\ b,\ c$ koeffitsiyentlarni toping.

20. $A(1;4)$, $B(-1;10)$, $C(2;7)$ nuqtalardan o'tuvchi $y=ax^2+bx+c$ funksiyani toping.

KVADRAT TENGSIZLIK

Ta'rif

Agar tengsizlikning chap qismida kvadrat uchhad, o'ng qismida esa nol turgan bo'lsa, bunday tengsizlik ***kvadrat* (bir noma'lumli ikkinchi darajali) *tengsizlik*** deyiladi.

$ax^2+bx+c>0$, $ax^2+bx+c<0$, $ax^2+bx+c\geq 0$, $ax^2+bx+c\leq 0$ $(a\neq 0)$ tengsizliklar kvadrat tengsizliklardir, bunda a, b, c – berilgan sonlar, x esa noma'lum son.

Tengsizlikning yechimi deb noma'lumning shu tengsizlikni to'g'ri sonli tengsizlikka aylantiruvchi barcha qiymatlari to'plamiga aytiladi.

Tengsizlikni yechish uning yechimini topish yoki yechimi yo'qligini ko'rsatish demakdir.

Kvadrat tengsizlikni quyidagi usullar bilan yechish mumkin:

1-usul. Chiziqli tengsizliklar sistemasiga keltirib yechish

Agar $ax^2+bx+c=0$ kvadrat tenglama ikkita turli ildizga ega bo'lsa, u holda kvadrat tengsizlikni yechishni birinchi darajali tengsizliklar sistemasini yechishga keltirish mumkin.

1-misol. $x^2-5x+6<0$ tengsizlikni yeching.

Yechish

Tengsizlikning chap tomonini ko'paytuvchilarga ajratamiz:

$$(x-2)(x-3)<0.$$

1-hol: $\begin{cases} x-2>0 \\ x-3<0 \end{cases} \Rightarrow \begin{cases} x>2 \\ x<3 \end{cases} \Rightarrow x\in(2;3).$ 2-hol: $\begin{cases} x-2<0 \\ x-3>0 \end{cases} \Rightarrow \begin{cases} x<2 \\ x>3 \end{cases} \Rightarrow x\in\varnothing.$

Javob: (2; 3).

2-usul. Kvadrat tengsizlikni kvadrat funksiya grafigi yordamida yechish

Kvadrat tengsizliklarni kvadrat funksiya grafigini yasab, grafik bo'yicha bu funksiya musbat yoki manfiy qiymatlarni qabul qiladigan oraliqlarni topib, yechish mumkin.

Kvadrat tengsizlikni grafik usulda yechish uchun:

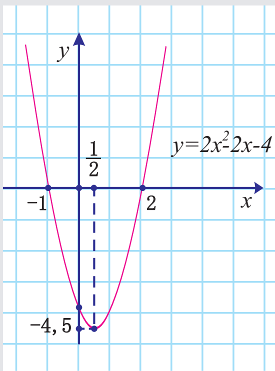
1) parabola tarmoqlari yo'nalishi aniqlanadi;

2) funksiya nollari (agar ular mavjud bo'lsa) topiladi yoki ularning yo'qligi aniqlanadi;

3) $y = ax^2 + bx + c$ funksiya grafigining eskizi chiziladi;

4) grafik bo'yicha funksiya musbat yoki manfiy qiymatlar qabul qiladigan oraliqlar ko'rsatiladi.

1-rasm



2-misol. $2x^2-2x-4\geq 0$ tengsizlikni kvadrat funksiya grafigi yordamida yeching.

Yechish. $y=2x^2-2x-4$ funksiya grafigini yasaymiz (1-*rasm*).

Avval parabola uchini topamiz:

$$x_0=-\frac{b}{2a}=-\frac{-2}{4}=\frac{1}{2};\quad y_0=2\left(\frac{1}{2}\right)^2-2\cdot\frac{1}{2}-4=-4,5.$$

Keyin diskriminantni hisoblab: $D = b^2 - 4ac = 4 + 32 = 36$, parabola nollarini topamiz:

$$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}=\frac{2\pm\sqrt{36}}{4}=\frac{2\pm 6}{4},$$

$x_1=-1$, $x_2=2$.

Javob: $(-\infty;-1]\cup[2;\infty)$.

Tengsizlikni bu usulda yechishda parabola uchining koordinatalarini topish shart emas, shuningdek, parabolaning *Oy* o'qi bilan kesishish nuqtalarining grafikda ko'rsatilishi muhim emas. Eng muhimi, parabola tarmoqlarining yo'nalishini va funksiya nollari bor yoki yo'qligini bilishdir.

3-usul. Kvadrat tengsizlikni oraliqlar (intervallar) usuli bilan yechish

Agar biror $(a;\ b)$ oraliqda $y = f(x)$ funksiya grafigini qalamni qog'ozdan uzmasdan chizish mumkin bo'lsa, bu funksiya $(a;\ b)$ oraliqda ***uzluksiz*** deyiladi.

Masalan, $y = kx + b$, $y = ax^2 + bx + c$ funksiyalar o'z aniqlanish sohasida uzluksiz funksiyalardir.

Uzluksiz funksiyalarning bir muhim xossasini isbotsiz qabul qilamiz.

Agar $f(x)$ funksiya $(a;\ b)$ oraliqda uzluksiz bo'lsa va nolga aylanmasa, u holda bu oraliqda funksiyaning qiymatlari bir xil ishoraga ega bo'ladi, ya'ni shu oraliqda funksiya o'z ishorasini saqlaydi.

Kvadrat funksiyaning aniqlanish sohasini uning x_1 va x_2 nollari yordamida uchta $(-\infty;\ x_1)$, $(x_1;\ x_2)$, $(x_2;\ \infty)$ oraliqlarga ajratish mumkin (bunda $x_1 < x_2$). Bu oraliqlarning har birida kvadrat funksiya uzluksiz va nolga aylanmaydi, ya'ni o'z ishorasini saqlaydi. Bir noma'lumli tengsizliklarni yechishning ***oraliqlar usuli*** aynan shu xossaga asoslangan.

Kvadrat tengsizliklarni yechishda oraliqlar usulining qo'llanishini ko'rib chiqamiz.

1-hol. $D > 0$. Bu holatda kvadrat funksiyaning nollari hisoblangan ikkita haqiqiy x_1 va x_2 $(x_1 < x_2)$ sonlar mavjud bo'ladi. Ular kvadrat funksiyaning aniqlanish sohasini: $(-\infty;\ x_1)$, $(x_1;\ x_2)$, $(x_2;\ \infty)$ oraliqlarga ajratadi va bu oraliqlarning har birida funksiyaning qiymatlari doimiy ishoraga ("+" yoki "–") ega bo'ladi.

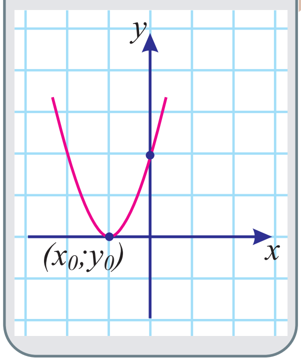
Kvadrat funksiya qiymatlarining hosil qilingan oraliqlardagi ishorasini turlicha usullar bilan topish mumkin:

1) $y = ax^2+bx+c$ funksiya qiymatining $(-\infty;\ x_1)$, $(x_2;\ \infty)$ oraliqlarning har biridagi ishorasi a koeffitsiyentning ishorasi bilan bir xil; $(x_1;\ x_2)$ oraliqdagi ishorasi esa a koeffitsiyent ishorasiga qarama-qarshi bo'ladi;

2) funksiya qiymatlarining ishorasini har bir oraliqdagi "qulay" nuqtada aniqlash mumkin;

3) $y = ax^2 + bx + c$ funksiyani $y = a(x - x_1)(x - x_2)$ ko'rinishda yozib, har bir oraliqda chiziqli ko'paytuvchilarning ishoralarini topish orqali aniqlash mumkin.

2-rasm



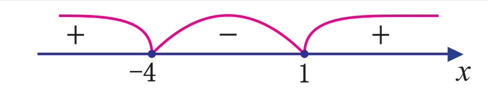
3-misol. $x^2 + 3x - 4 \le 0$ tengsizlikni oraliqlar usuli bilan yeching.

Yechish. Tengsizlikning chap qismini ko'paytuvchilarga ajratamiz:

$$(x+4)(x-1) \le 0.$$

Uning nollari: – 4 va 1.

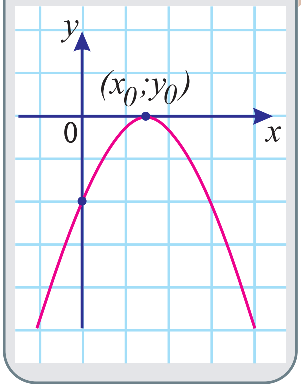
Topilgan nuqtalarni son o'qida belgilaymiz va o'qni oraliqlarga ajratamiz. Har bir oraliqda $y = x^2 + 3x - 4$ funksiyaning ishorasini aniqlaymiz:



Berilgan misol shartida funksiya o'zining musbat bo'lmagan qiymatlariga qaysi oraliqda erishishi so'ralgani uchun yechim [–4; 1] bo'ladi.

Javob: [–4; 1].

3-rasm

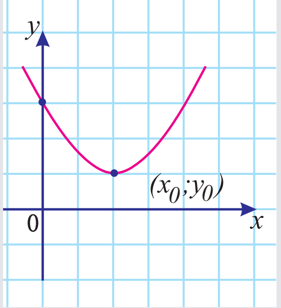


2-hol. $D = 0$ bo'lsin. U holda $y = ax^2 + bx + c$ funksiya faqat bitta x_0 nuqtada nolga aylanadi. x_0 nuqta koordinata o'qini ikkita: $(-\infty; x_0)$ va $(x_0; \infty)$ oraliqlarga ajratadi. Har bir $x \ne x_0$ uchun $y = ax^2 + bx + c$ kvadrat funksiya qiymatlarining ishorasi a koeffitsiyentning ishorasi bilan bir xil bo'ladi (2, 3-*rasmlar*).

3-hol. $D < 0$. U holda $y = ax^2 + bx + c$ kvadrat funksiya nollarga ega bo'lmaydi.

Bu holatda x ning ixtiyoriy qiymatlarida funksiya a koeffitsiyentning ishorasi bilan ustma-ust tushadigan bir xil ishorali qiymatlarni qabul qiladi:

4-rasm



1) agar $a > 0$ bo'lsa, x ning ixtiyoriy qiymatida $ax^2 + bx + c > 0$;

2) agar $a < 0$ bo'lsa, x ning ixtiyoriy qiymatida $ax^2 + bx + c < 0$.

Quyidagilarni bilish va qo'llay olish muhim:

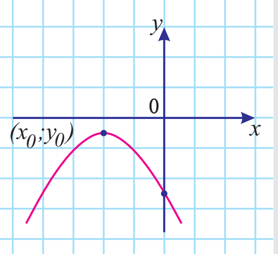
1) $a > 0$ va $D < 0$ bo'lganda $ax^2 + bx + c > 0$, $ax^2 + bx + c \ge 0$ tengsizliklarning yechimlari barcha haqiqiy sonlar to'plamidan iborat (4-*rasm*);

2) $a > 0$ va $D < 0$ bo'lganda $ax^2 + bx + c < 0$, $ax^2 + bx + c \le 0$ tengsizliklarning yechimlari bo'sh to'plamdan iborat (4-*rasm*);

3) $a < 0$ va $D < 0$ bo'lganda $ax^2 + bx + c > 0$, $ax^2 + bx + c \ge 0$ tengsizliklarning yechimlari bo'sh to'plamdan iborat (5-*rasm*);

4) $a < 0$ va $D < 0$ bo'lganda $ax^2 + bx + c < 0$, $ax^2 + bx + c \le 0$ tengsiz- liklarning yechimlari barcha haqiqiy sonlar to'plamidan iborat bo'ladi (5-*rasm*).

5-rasm



MISOLLAR

1. 0; −2; 3 sonlaridan qaysilari $-4x^2+5x-5>0$ tengsizlikni qanoatlantiradi?

2. Quyidagi tengsizliklarni chiziqli tengsizliklar sistemasiga keltirib yeching.

a) $(x+4)(2x-3)>0$ b) $x^2+10x-11<0$

c) $(5x-2)(4x+3)\le 0$ d) $2x^2-5x+2\ge 0$

3. Tengsizliklar teng kuchlimi?

a) $5x^2 > 2x$ va $5x > 2$ b) $3x^3 < 7x^2$ va $3x < 7$ c) $\dfrac{x^2-1}{x}>0$ va $(x^2-x)(x+1)>0$

4. Tengsizliklarni yeching.

a) $x^2>0$ b) $4x^2\ge 0$ c) $x^2<0$

d) $-x^2\le 0$ e) $x^2+7>0$ f) $5x^2+11\le 0$

g) $-x^2-5>0$ h) $3x^2-2x<0$ i) $-4x^2+11x<0$

j) $x^2-9x+20<0$ k) $x^2-10x+25>0$ l) $-x^2+6x-8>0$

m) $3x^2-x+2\ge 0$ n) $-9x^2+24x+20>0$ o) $-7\cdot(3-x)^2>0$

5. Yechimi berilgan oraliqlardan iborat bo'lgan biror kvadrat tengsizlik tuzing.

a) $(-\infty;-3)\cup(6;\infty)$ b) $(-\infty;\ \infty)$

6. Abssissa o'qida $x^2+9x\le -14$ tengsizlikning yechimi bo'lgan kesma uzunligini toping.

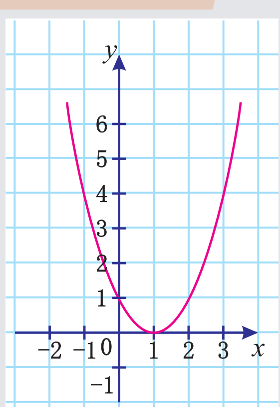
7. Nechta butun son $2x^2+7x-15<0$ tengsizlikni qanoatlantiradi?

8. Tengsizlikni oraliqlar usuli bilan yeching.

a) $x^2+5x-6>0$ b) $-x^2+x+2<0$ c) $x^2+3x+7>0$ d) $x^2+3x+7\le 0$

e) $-2x^2+5x+3>0$ f) $6x^2-x-2<0$ g) $2x^2+5x+9\le 0$ h) $49x^2-28x+4\le 0$

6-rasm



9. Tengsizliklarni yeching.

a) $8x^2+3x-5\ge 0$ b) $5x^2-12x+8\le 0$

c) $49x^2-70x+25>0$ d) $(2x^2+3x+4)(x+3)\ge 0$

e) $(7+6x-x^2)(3x-5)<0$

10. Tengsizlikni kvadrat funksiya grafigi yordamida yeching.

a) $2x^2+5x-3>0$ b) $4x^2-9x-90>0$

11. 6-rasmda $y = ax^2 + bx + c$ funksiya grafigi tasvirlangan. Quyidagi tengsizliklarning yechimini toping.

a) $ax^2+bx+c>0$ b) $ax^2+bx+c\le 0$

12. Tengsizlikning barcha butun yechimlari yig'indisini toping.

a) $2x^2-9x+4<0$ b) $\frac{x-1}{4}+\frac{3-2x}{2}>\frac{3x+x^2}{8}$

c) $(5x+7)(x-2)\le 21x^2-11x+3$

13. $3x(x-2)<2x(x+4)-(x-16)\le 0$ tengsizlikning $[0;9]$ kesmaga tegishli bo'lgan nechta butun yechimi bor?

14. $y=-x^2+4x-3$ funksiya grafigi yordamida quyidagi tengsizliklarning yechimini toping.

a) $-x^2+4x-3>0$ b) $-x^2+4x-3\ge 0$ c) $-x^2+4x-3<0$ d) $-x^2+4x-3\le 0$

15. a ning qanday qiymatlarida $ax^2+2ax+4=0$ tenglama ildizlarga ega bo'lmaydi?

16. Tengsizlikni yeching: $(x-1)^2(x^2-2)<(x-1)^2(6-2x)$

17. $f(x)=(x-1)^4(x+1)^3x^2$ funksiya berilgan.

a) $f(x)<0$ b) $f(x)\le 0$ c) $f(x)>0$ d) $f(x)\ge 0$

bo'ladigan x ning barcha qiymatlarini toping.

18. Tengsizliklarni yeching:

a) $x^2-2(b-c)x+a^2>0$, bunda a, b, c lar uchburchakning tomonlari;

b) $x^2+(a^2+b^2-c^2)x+a^2b^2>0$, bunda a, b, c lar uchburchakning tomonlari.

19. Agar $a^2+12b<0$ bo'lsa, $3x^2-b\le ax$ ni yeching.

20. Agar $b>0{,}05a^2$ bo'lsa, $5x^2-ax+b>0$ ni yeching.

21. Agar $b^2\le 4ac$ va $a+c>b$ bo'lsa, $ax^2+bx+c\le 0$ ni yeching.

22. c ning qanday qiymatlarida $y=cx^2+x+c$ va $y=cx+1-c$ funksiyalar grafiklari umumiy nuqtaga ega bo'lmaydi?

23. p ning qanday qiymatlarida $y=px^2-24x+1$ va $y=12x^2-2px-1$ funksiyalar grafigi kesishmaydi?

24. a ning qanday qiymatlarida $x^2+3x+a=0$ tenglamaning ildizlari $\frac{x_1}{x_2}+\frac{x_2}{x_1}+1>0$ shartni qanoatlantiradi?

25. b ning qanday qiymatlarida $x^2-2bx+b+6=0$ tenglamaning:

a) ildizlari manfiy; b) ildizlari musbat; c) ildizlari turli ishorali bo'ladi?

26. a ning qanday qiymatlarida barcha haqiqiy sonlar tengsizlikni qanoatlantiradi?

a) $x^2-(a+2)x+8a+1>0$ b) $\frac{1}{24}x^2+ax-a+1>0$

27. b ning qanday qiymatlarida tengsizlik yechimga ega emas?

a) $x^2+2bx+1<0$ b) $bx^2+4bx+5<0$ c) $bx^2+(2b+3)x+b-1\ge 0$

TRIGONOMETRIK AYNIYATLAR

Asosiy trigonometrik ayniyatlar

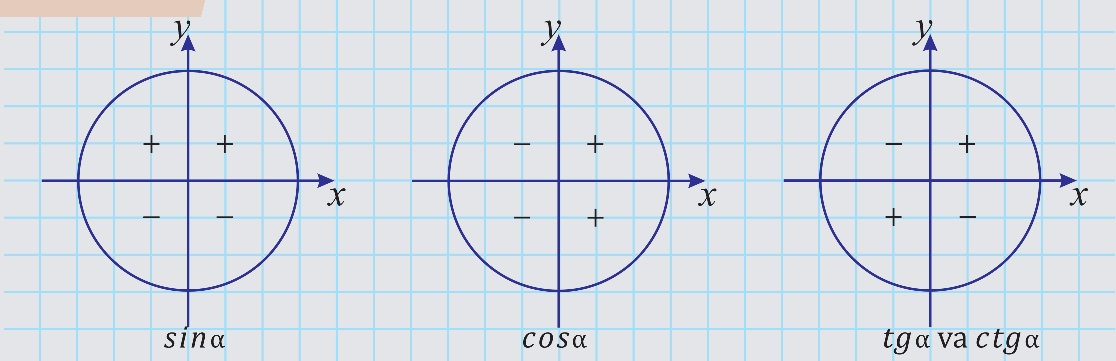
1. $\sin^2\alpha + \cos^2\alpha = 1$
2. $tg\alpha = \dfrac{\sin\alpha}{\cos\alpha}, \quad \cos\alpha \neq 0$
3. $ctg\alpha = \dfrac{\cos\alpha}{\sin\alpha}, \quad \sin\alpha \neq 0$
4. $tg\alpha \cdot ctg\alpha = 1$
5. $1 + tg^2\alpha = \dfrac{1}{\cos^2\alpha}, \quad \cos\alpha \neq 0$
6. $1 + ctg^2\alpha = \dfrac{1}{\sin^2\alpha}, \quad \sin\alpha \neq 0$

Ba'zi burchaklarning sinusi, kosinusi, tangensi va kotangensining qiymatlari

α	$0^0\,(0)$	$30°\left(\frac{\pi}{6}\right)$	$45°\left(\frac{\pi}{4}\right)$	$60°\left(\frac{\pi}{3}\right)$	$90°\left(\frac{\pi}{2}\right)$	$180°(\pi)$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	–1
$tg\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Mavjud emas	0
$ctg\alpha$	Mavjud emas	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	Mavjud emas

Sinus, kosinus, tangens va kotangensning ishoralari

1-rasm



α va (–α) burchaklarning sinusi, kosinusi, tangensi va kotangensi

1. $\sin(-\alpha) = -\sin\alpha$
2. $\cos(-\alpha) = \cos\alpha$
3. $tg(-\alpha) = -tg\alpha$
4. $ctg(-\alpha) = -ctg\alpha$

Keltirish formulalari

	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$	$2\pi+\alpha$
$sin\,\alpha$	$cos\,\alpha$	$cos\,\alpha$	$sin\,\alpha$	$-sin\,\alpha$	$-cos\,\alpha$	$-cos\,\alpha$	$-sin\,\alpha$	$sin\,\alpha$
$cos\,\alpha$	$sin\,\alpha$	$-sin\,\alpha$	$-cos\,\alpha$	$-cos\,\alpha$	$-sin\,\alpha$	$sin\,\alpha$	$cos\,\alpha$	$cos\,\alpha$
$tg\,\alpha$	$ctg\,\alpha$	$-ctg\,\alpha$	$-tg\,\alpha$	$tg\,\alpha$	$ctg\,\alpha$	$-ctg\,\alpha$	$-tg\,\alpha$	$tg\,\alpha$
$ctg\,\alpha$	$tg\,\alpha$	$-tg\,\alpha$	$-ctg\,\alpha$	$ctg\,\alpha$	$tg\,\alpha$	$-tg\,\alpha$	$-ctg\,\alpha$	$ctg\,\alpha$

Keltirish formulalaridagi ushbu qonuniyatga e'tibor qarating: agar α ni I chorakka tegishli deb olsak, $\pi\pm\alpha$, $2\pi\pm\alpha$ burchaklar uchun keltirish formulalarida funksiya nomi almashmaydi, $\frac{\pi}{2}\pm\alpha$, $\frac{3\pi}{2}\pm\alpha$ burchaklar uchun esa sinus kosinusga, kosinus sinusga, tangens kotangensga, kotangens tangensga almashadi.

Masalan, $sin\left(n\cdot\frac{\pi}{2}\pm\alpha\right)$ ni qarasak, $\frac{\pi}{2}$ lar sonini ifodalaydigan n – natural son juft bo'lsa, funksiya nomi almashmaydi; toq bo'lsa, funksiya nomi almashadi. Ishorani aniqlash esa $n\cdot\frac{\pi}{2}\pm\alpha$ burchak qaysi chorakka tegishli ekani va bu chorakda sinusning ishorasi qandayligiga qarab aniqlanadi.

1-misol. Hisoblang.

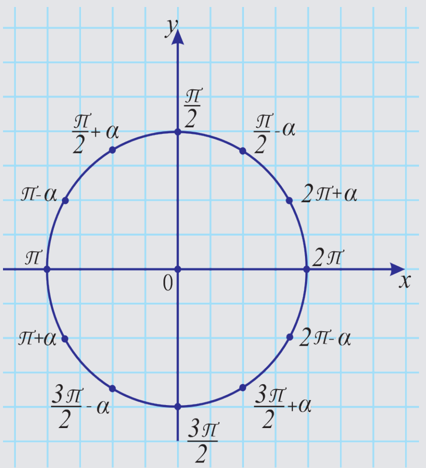
a) $sin855^\circ = sin(9\cdot90^\circ+45^\circ) = cos45^\circ = \frac{\sqrt{2}}{2}$

b) $cos2025^\circ = cos(22\cdot90^\circ+45^\circ) = -\,cos45^\circ = -\frac{\sqrt{2}}{2}$

c) $tg1680^\circ = tg(18\cdot90^\circ+60^\circ) = tg60^\circ = \sqrt{3}$

d) $ctg1200^\circ = ctg(13\cdot90^\circ+30^\circ) = -tg30^\circ = -\frac{\sqrt{3}}{3}$

2-rasm



Qo‘shish formulalari

1. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
2. $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$
3. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
4. $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
5. $tg(\alpha+\beta)=\dfrac{tg\alpha+tg\beta}{1-tg\alpha tg\beta}$
6. $tg(\alpha-\beta)=\dfrac{tg\alpha-tg\beta}{1+tg\alpha tg\beta}$
7. $ctg(\alpha+\beta)=\dfrac{ctg\alpha ctg\beta-1}{ctg\alpha+ctg\beta}$
8. $ctg(\alpha-\beta)=\dfrac{ctg\alpha ctg\beta+1}{ctg\beta-ctg\alpha}$

Ikkilangan burchak formulalari

1. $\sin 2\alpha=2\sin\alpha\cdot\cos\alpha$
2. $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=2\cos^2\alpha-1=1-2\sin^2\alpha$
3. $tg2\alpha=\dfrac{2tg\alpha}{1-tg^2\alpha}$
4. $ctg2\alpha=\dfrac{ctg^2\alpha-1}{2ctg\alpha}$

Yig‘indi va ayirmani ko‘paytmaga almashtirish formulalari

1. $\sin\alpha+\sin\beta=2\sin\dfrac{\alpha+\beta}{2}\cdot\cos\dfrac{\alpha-\beta}{2}$
2. $\sin\alpha-\sin\beta=2\sin\dfrac{\alpha-\beta}{2}\cdot\cos\dfrac{\alpha+\beta}{2}$
3. $\cos\alpha+\cos\beta=2\cos\dfrac{\alpha+\beta}{2}\cdot\cos\dfrac{\alpha-\beta}{2}$
4. $\cos\alpha-\cos\beta=-2\sin\dfrac{\alpha+\beta}{2}\cdot\sin\dfrac{\alpha-\beta}{2}$
5. $tg\alpha+tg\beta=\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$
6. $tg\alpha-tg\beta=\dfrac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$
7. $ctg\alpha+ctg\beta=\dfrac{\sin(\alpha+\beta)}{\sin\alpha\cdot\sin\beta}$
8. $ctg\alpha-ctg\beta=\dfrac{-\sin(\alpha-\beta)}{\sin\alpha\cdot\sin\beta}$

Ko‘paytmani yig‘indiga almashtirish formulalari

1. $\sin\alpha\cdot\cos\beta=\dfrac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$
2. $\cos\alpha\cdot\cos\beta=\dfrac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta))$
3. $\sin\alpha\cdot\sin\beta=\dfrac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta))$

Daraja pasaytirish formulalari

1. $\sin^2\alpha=\dfrac{1-\cos 2\alpha}{2}$
2. $\cos^2\alpha=\dfrac{1+\cos 2\alpha}{2}$
3. $tg^2\alpha=\dfrac{1-\cos 2\alpha}{1+\cos 2\alpha}$
4. $ctg^2\alpha=\dfrac{1+\cos 2\alpha}{1-\cos 2\alpha}$

Yarim burchak formulalari

1. $\cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}$
2. $\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}$
3. $tg^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{1+\cos\alpha}$
4. $ctg^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{1-\cos\alpha}$
5. $tg\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}$
6. $ctg\frac{\alpha}{2}=\frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}$

$\sin\alpha$, $\cos\alpha$, $tg\alpha$ larni $tg\frac{\alpha}{2}$ orqali ifodalash formulalari

1. $\sin\alpha=\dfrac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$
2. $\cos\alpha=\dfrac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$
3. $tg\alpha=\dfrac{2tg\frac{\alpha}{2}}{1-tg^2\frac{\alpha}{2}}$

MISOLLAR

1. Agar $ctg\alpha=-\frac{3}{4}$ va $\frac{3\pi}{2}<\alpha<2\pi$ bo'lsa, $\cos\alpha$ ni toping.

2. Agar $tg\alpha=-\sqrt{5}$ va $\frac{\pi}{2}<\alpha<\pi$ bo'lsa, $\sin\alpha$ ni toping.

3. Agar $tg\alpha=\frac{3}{2}$ bo'lsa, $\frac{2\sin\alpha+5\cos\alpha}{3\sin\alpha-4\cos\alpha}$ ni toping.

4. Soddalashtiring.

a) $\frac{2\sin^2\alpha-1}{2\cos^2\alpha-1}$ b) $\frac{ctg\left(2\sin^2\alpha+tg\alpha\right)}{\left(\sin\alpha+\cos\alpha\right)^2}$ c) $\frac{1-\cos^2\alpha}{\sin^4\alpha}-ctg^2\alpha$ d) $2-\frac{1-\sin^4\alpha}{\cos^2\alpha}$

5. Hisoblang.

a) $4\cos150°-\sin240°-3tg210°$ b) $2\cos135°+tg60°-ctg240°$

c) $\sin300°-3\cos135°+2\cos210°$ d) $tg150°-ctg315°+5\sin135°$

6. Hisoblang.

a) $tg\frac{7\pi}{6}-2tg\frac{5\pi}{3}+3tg\frac{11\pi}{6}$ b) $2\cos\frac{4\pi}{3}+\sin\frac{5\pi}{4}-\cos\frac{4\pi}{3}$

c) $20ctg\frac{3\pi}{2}-\sin\frac{2\pi}{3}+\cos\frac{4\pi}{3}$ d) $ctg\frac{7\pi}{4}+2ctg\frac{4\pi}{3}-2\cos\frac{5\pi}{6}$

7. Soddalashtiring.

a) $\frac{1-tg(360°-\alpha)tg\alpha}{tg(270°-\alpha)+tg\alpha}$ b) $\frac{\cos(90°+\alpha)+\sin(90°-\alpha)}{\cos(270°-\alpha)+\cos(180°+\alpha)}$

8. Soddalashtiring.

a) $\dfrac{sin\left(\dfrac{\pi}{2}+\beta\right)}{tg^2\left(\dfrac{\pi}{2}-\dfrac{\beta}{2}\right)-ctg^2\left(\dfrac{\pi}{2}+\dfrac{\beta}{2}\right)}$ b) $\dfrac{cos\left(\dfrac{3\pi}{2}-2a\right)+sin\left(\dfrac{3\pi}{2}+2a\right)}{cos\left(\dfrac{3\pi}{2}+2a\right)+sin\left(\dfrac{3\pi}{2}-2a\right)}$

9. Ayniyatni isbotlang.

a) $\dfrac{sin(\pi-2\alpha)-2sin\left(\dfrac{\pi}{2}-\alpha\right)}{sin^2(\pi+\alpha)-cos\left(\dfrac{3\pi}{2}+\alpha\right)}=2ctg\alpha$ b) $\dfrac{sin^4\left(\alpha-\dfrac{\pi}{2}\right)-cos^2(2\alpha+\pi)}{1-3cos(2\alpha+\pi)}=\dfrac{sin^2\alpha}{2}$

10. Hisoblang.

a) $sin(-43°)cos88°+cos(-43°)sin88°$ b) $cos11°cos19°-sin19°sin11°$

11. Hisoblang.

a) $sin\dfrac{2\pi}{7}cos\dfrac{3\pi}{14}+cos\dfrac{2\pi}{7}sin\dfrac{3\pi}{14}$ b) $\dfrac{1+tg33°tg78°}{tg78°-tg33°}$

12. Hisoblang.

a) $cos\left(-\dfrac{19\pi}{36}\right)cos\dfrac{7\pi}{9}-sin\dfrac{7\pi}{9}sin\left(-\dfrac{19\pi}{36}\right)$ b) $\dfrac{1-tg\dfrac{\pi}{11}tg\dfrac{5\pi}{66}}{tg\dfrac{5\pi}{66}+tg\dfrac{\pi}{11}}$

13. Soddalashtiring.

a) $cos(\alpha-\beta)-sin\alpha sin\beta$ b) $sin(\alpha-\beta)+sin(\alpha+\beta)$

c) $sin4\alpha\, cos\alpha-cos4\alpha\, sin\alpha$ d) $cos\alpha\, cos2\alpha+sin2\alpha\, sin\alpha$

e) $\dfrac{1}{2}cos\alpha+\dfrac{\sqrt{3}}{2}sin\alpha$ f) $\dfrac{1}{\sqrt{2}}cos\alpha-\dfrac{\sqrt{2}}{2}sin\alpha$

14. a) $cos\alpha=\dfrac{2\sqrt{2}}{3}$ va $\dfrac{3\pi}{2}<\alpha<2\pi$ bo'lsa, $cos\left(\alpha+\dfrac{\pi}{4}\right)$ ni toping.

b) $cos\alpha=-\dfrac{1}{\sqrt{3}}$ va $\pi<\alpha<\dfrac{3\pi}{2}$ bo'lsa, $sin\left(\alpha+\dfrac{\pi}{6}\right)$ ni toping.

15. Hisoblang.

a) $\dfrac{6sin10°cos10°}{sin20°}$ b) $\dfrac{sin88°}{sin22°cos22°cos44°}$ c) $sin\dfrac{\pi}{12}\left(2sin^2\dfrac{\pi}{24}-1\right)$

16. a) Agar $cos\alpha=0,4$ bo'lsa, $cos2\alpha$ ni toping.

b) Agar $sin\alpha=-0,7$ bo'lsa, $cos2\alpha$ ni toping.

17. a) Agar $cos\alpha = \frac{2\sqrt{2}}{3}$ va $\frac{3\pi}{2} < \alpha < 2\pi$ bo'lsa, $sin2\alpha$ ni toping.

b) Agar $sin\alpha = \frac{1}{5}$ va $\frac{\pi}{2} < \alpha < \pi$ bo'lsa, $sin2\alpha$ ni toping.

18. Soddalashtiring:

a) $2cos^2\frac{\alpha}{2}(cos\,\alpha - 1)$ b) $\frac{sin2\alpha}{1+cos2\alpha}$

19. Agar $tg\alpha = -2$ bo'lsa, $sin2\alpha,\ cos2\alpha,\ tg2\alpha,\ ctg2\alpha$ larni toping.

20. a) $cos123°tg231°sin312°$ ifodaning ishorasini aniqlang.

b) $sin\frac{1}{3}cos\frac{7}{8}tg4ctg5{,}7$ ifodaning ishorasini aniqlang.

21. Sonlarni taqqoslang: $sin200°$ va $sin(-200°)$.

22. $sin\,\alpha = \frac{12}{13}$ va $cos\,\alpha = \frac{5}{13}$ tengliklarning ikkalasi bir vaqtda o'rinli bo'lishi mumkinmi?

23. Ayniyatni isbotlang.

a) $\left(sin\,\alpha + \frac{1}{sin\,\alpha}\right)^2 + \left(cos\,\alpha + \frac{1}{cos\,\alpha}\right)^2 - \left(tg^2\alpha + ctg^2\alpha\right) = 7$ b) $\frac{tg3\alpha}{tg^2 3\alpha - 1}\cdot\frac{1-ctg^2 3\alpha}{ctg3\alpha} = 1$

24. Ifodani soddalashtiring.

a) $\frac{1-ctg^2(-\alpha)}{cos\,\alpha + sin(-\alpha)}\cdot sin(-\alpha) + ctg(-\alpha)$ b) $\frac{sin(\alpha-\beta) - sin(\beta-\alpha)}{cos(\alpha-\beta) + cos(\beta-\alpha)}$

25. Ayniyatni isbotlang:

a) $sin\alpha + cos\alpha = \sqrt{2}\,cos\left(\alpha - \frac{\pi}{4}\right)$ b) $2sin2\alpha cos5\alpha = sin7\alpha - sin3\alpha$.

26. $cos\,\alpha = \frac{2}{3};\ sin\beta = -\frac{\sqrt{7}}{4};\ \frac{3\pi}{2} < \alpha < 2\pi;\ \pi < \beta < \frac{3\pi}{2}$ bo'lsa, $cos(\alpha+\beta)$ ni toping.

27. Ayniyatni isbotlang: $\frac{tg\alpha - tg\beta}{tg\alpha + tg\beta} = \frac{sin(\alpha-\beta)}{sin(\alpha+\beta)}$.

28. Hisoblang: $sin(-300^0)cos(-135^0)tg(-210^0)ctg(-120^0)$.

29. Agar $sin\,\alpha + cos\,\alpha = \frac{2}{3}$ bo'lsa, $sin\alpha cos\alpha$ ni toping.

30. Agar $sin\,\alpha\,cos\,\alpha = \frac{1}{3}$ va $\pi < a < \frac{3\pi}{2}$ bo'lsa, $sin\alpha + cos\alpha$ ni toping.

ARIFMETIK VA GEOMETRIK PROGRESSIYA

Arifmetik progressiya

1. $a_{n+1} = a_n + d$, $n \in N$;
2. $a_n = a_1 + (n-1)d$, $n \in N$;
3. $a_n = a_k + (n-k)d$, $n, k \in N$, $n > k$;
4. $a_n = \dfrac{a_{n-1} + a_{n+1}}{2}$, $n \in N$;
5. $a_n = \dfrac{a_{n-k} + a_{n+k}}{2}$, $n, k \in N$, $n > k$;
6. $\{a_n\}$ arifmetik progressiya hadlari uchun $a_n + a_m = a_k + a_l$ tenglik o‘rinli, bunda $n + m = k + l$;
7. $S_n = \dfrac{(a_1 + a_n)n}{2}$;
8. $S_n = \dfrac{(2a_1 + (n-1)d)n}{2}$.

Geometrik progressiya

1. $b_{n+1} = b_n \cdot q$, $n \in N$;
2. $b_n = b_1 \cdot q^{n-1}$, $n \in N$;
3. $b_n = b_k \cdot q^{n-k}$, $n, k \in N$ va $n > k$;
4. $b_n^2 = b_{n-k} \cdot b_{n+k}$, $n, k \in N$, $n > k$;
5. $\{b_n\}$ arifmetik progressiya hadlari uchun $b_n \cdot b_m = b_k \cdot b_l$ tenglik o‘rinli, bunda $n + m = k + l$;
6. $S_n = \dfrac{b_1(1-q^n)}{1-q}$, $q \neq 1$, agar $q = 1$ bo‘lsa, $S_n = b_1 \cdot n$;
7. $S_n = \dfrac{b_n q - b_1}{q-1}$, $q \neq 1$;
8. Cheksiz kamayuvchi geometrik progressiya (barcha hadlari) yig‘indisi:

$$S = \frac{b_1}{1-q}, \quad |q| < 1, q \neq 0.$$

MISOLLAR

1. Agar $a_1 = -3$ va $d = 6$ bo'lsa, arifmetik progressiyaning saksoninchi hadini toping.

2. 2, 6, 10, 14, 18, ... ketma-ketlik arifmetik progressiya tashkil qiladi. Uning n-hadi formulasini yozing.

3. Arifmetik progressiyada:

a) $a_7 = -5$, $a_{32} = 70$ bo'lsa, a_1 va d ni;

b) $a_5 = 2$, $a_{40} = 142$ bo'lsa, a_7 ni;

c) $a_{14} = 5$, $a_{12} = 1$ bo'lsa, a_{13} ni;

d) $a_{25} - a_{20} = 10$, $a_{16} = 13$ bo'lsa, a_{10} ni toping.

4. Agar geometrik progressiyada $b_2 = 4$ va $b_3 = 6$ bo'lsa, b_7 ni toping.

5. Agar geometrik progressiyada $b_1 = 3$ va $q = -2$ bo'lsa, b_8 ni toping.

6. Geometrik progressiyada:

a) $b_1 = 18$, $q = \frac{1}{9}$ bo'lsa, b_2 ni;

b) $b_1 = \frac{1}{2}$, $q = \frac{1}{2}$ bo'lsa, b_7 ni;

c) $b_4 = 8$, $b_8 = 128$ bo'lsa, b_1 va q ni;

d) $b_9 = -1$, $q = -1$ bo'lsa, b_1 va b_{17} ni toping.

7. Geometrik progressiyada $b_1 = 3$, $q = 2$ bo'lsa, S_6 ni toping.

8. Geometrik progressiyada $b_2 = 6$, $q = 3$ bo'lsa, S_8 ni toping.

9. Geometrik progressiyada $b_1 = 4$, $q = \frac{1}{2}$ bo'lsa, dastlabki 10 ta hadi yig'indisini toping.

10. Geometrik progressiyaning birinchi hadi 5, oltinchi hadi 1215 ga teng. Shu progressiya maxrajini toping.

11. Cheksiz kamayuvchi geometrik progressiyada $b_1 = 8$, $q = \frac{1}{2}$ bo'lsa, uning yig'indisini toping.

12. 12, 4, $\frac{4}{3}$, ... cheksiz kamayuvchi geometrik progressiya yig'indisini toping.

13. Geometrik progressiyada:

a) $b_1 = 24$, $b_2 = 36$ bo'lsa, q ni;

b) $b_5 = 36$, $b_7 = 144$ bo'lsa, b_6 ni;

c) $b_6 = \frac{1}{486}$, $b_8 = \frac{1}{4374}$ bo'lsa, b_7 ni toping.

14. Cheksiz kamayuvchi geometrik progressiya yig'indisi 150 ga teng. Agar $q = \frac{1}{3}$ bo'lsa, b_1 ni toping.

15. Cheksiz kamayuvchi geometrik progressiyada $b_1 = \frac{1}{4}$, $S = 16$ bo'lsa, q ni toping.

16. Geometrik progressiyada:

a) $b_1 = 3$, $q = 5$ bo'lsa, S_4 ni;

b) $b_2 = 8$, $b_3 = 4$ bo'lsa, S_6 ni;

c) $b_1 = -2$, $b_6 = -486$ bo'lsa, S_6 ni toping.

17. Geometrik progressiyada $q=-\frac{1}{2}$, $S_8=\frac{85}{64}$ bo'lsa, b_1 ni toping.

18. Arifmetik progressiyaning n-hadining formulasini yozing.

a) $a_1=5,\ a_2=-5$ b) $a_1=-3,\ a_6=12$ c) $a_1=6,\ a_{10}=33$

19. Arifmetik progressiyaning to'rtinchi va oltinchi hadlari mos ravishda 16 va 19 ga teng bo'lsa, birinchi hadini toping.

20. Dastlabki 25 ta natural son yig'indisini toping.

21. (a_n) arifmetik progressiyada $a_3+a_7=5$ va $a_4=1$ bo'lsa, uning dastlabki o'nta hadi yig'indisini toping.

22. Arifmetik progressiyada $a_1=-20{,}7$, $d=1{,}8$ bo'lsa, nechanchi hadidan boshlab progressiyaning barcha hadlari musbat bo'ladi?

23. Arifmetik progressiyada $a_{12}+a_{15}=20$ bo'lsa, S_{26} ni toping.

24. Arifmetik progressiyada $a_2+a_6=44$, $a_5-a_1=20$ bo'lsa, a_{100} ni toping.

25. Arifmetik progressiyaning uchinchi va to'qqizinchi hadlari yig'indisi 8 ga teng. Shu progressiyaning dastlabki o'n bitta hadi yig'indisini toping.

26. Arifmetik progressiyada $S_n=3n^2+n$ bo'lsa, a_1 va d ni toping.

27. O'suvchi geometrik progressiyada $b_{12}=4\cdot b_{10}$ va $b_3=6$ bo'lsa, b_7 ni toping.

28. Geometrik progressiyada $b_5=\sqrt[3]{2}$. Shu progressiya dastlabki to'qqizta hadi ko'paytmasini toping.

29. Geometrik progressiyada $S_4=10\frac{5}{8}$, $S_5=42\frac{5}{8}$, $b_1=\frac{1}{8}$ bo'lsa, q ni toping.

30. Geometrik progressiyada $b_1=1$ va $b_3+b_5=90$ bo'lsa, q ni toping.

31. Uchta x, y va 12 soni kamayuvchi geometrik progressiyani tashkil qiladi. Agar 12 ning o'rniga 9 qo'yilsa, arifmetik progressiya hosil bo'ladi. $x+y$ ni toping.

32. Geometrik progressiyada $b_2\cdot b_4\cdot b_6=216$ va $b_3=12$. Shu progressiyaning dastlabki oltita hadi yig'indisini toping.

33. Cheksiz kamayuvchi geometrik progressiyaning toq o'rinlarda turgan hamma hadlarining yig'indisi 36 ga teng. Juft o'rinlarda turgan hamma hadlarining yig'indisi 12 ga teng. Shu progressiya maxrajini va ikkinchi hadini toping.

34. Cheksiz kamayuvchi geometrik progressiyaning birinchi va to'rtinchi hadlarining yig'indisi 54 ga, ikkinchi va uchinchi hadlarining yig'indisi 36 ga teng. Shu progressiyaning yig'indisini toping.

35. Arifmetik progressiyada:

a) $a_1=-3$, $a_3\cdot a_7=24$ bo'lsa, S_{12} ni; b) $a_2+a_9=20$ bo'lsa, S_{10} ni;

c) $a_3+a_6=19$ bo'lsa, S_8 ni; d) $S_4=-28$, $S_6=58$ bo'lsa, S_{16} ni toping.

1-BOB. FUNKSIYALAR

- FUNKSIYA. FUNKSIYANING BERILISH USULLARI
- FUNKSIYANING ANIQLANISH SOHASI VA QIYMATLAR TOʻPLAMI
- FUNKSIYALAR USTIDA ARIFMETIK AMALLAR
- MURAKKAB, TESKARI, DAVRIY FUNKSIYALAR
- FUNKSIYA XOSSALARI
- FUNKSIYA GRAFIGI USTIDA SODDA ALMASHTIRISHLAR
- CHIZIQLI VA KVADRATIK MODELLASHTIRISHLAR
- LOYIHA ISHI

FUNKSIYA. FUNKSIYANING BERILISH USULLARI

Funksiya

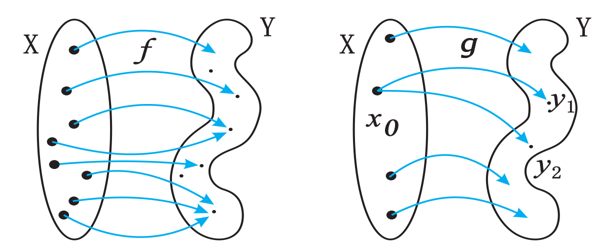
Tabiat, ishlab chiqarish, iqtisodiyot va boshqa sohalarda ayrim miqdorlar orasidagi bog‘lanishlarni o‘rganishda ***funksiya*** tushunchasi muhim ahamiyatga ega.

X va Y sonli to‘plamlar bo‘lsin. Har bir $x \in X$ nuqtaga yagona $y \in Y$ nuqtani mos qo‘yuvchi qonuniyat ***funksiya*** deyiladi.

Funksiyani aniqlovchi qonuniyatlar f, g, ... harflari orqali belgilanadi. $y = f(x)$ yozuv f qonuniyat $x \in X$ nuqtaga $y \in Y$ nuqtani mos qo‘yganini anglatadi va bu holda X to‘plamning nuqtalarini Y to‘plamning nuqtalariga mos qo‘yuvchi f funksiya berilgan deyiladi. Bunda, x ***erkli o‘zgaruvchi*** yoki ***argument***, y esa ***erksiz o‘zgaruvchi*** yoki ***funksiya*** deb yuritiladi. f funksiya odatda $y = f(x)$ yoki $f(x)$ ko‘rinishlarda ifodalanadi.

Quyida ayrim funksiyalar keltirilgan:

1-rasm



f qonuniyat funksiya bo‘ladi: X ning har bir x elementiga Y dan yagona y element mos qo‘yilgan.

g qonuniyat funksiya emas: $x_0 \in X$ elementga ikkita $y_1, y_2 \in Y$ elementlar mos qo‘yilgan.

Funksiya bo‘ladigan (f) va funksiya bo‘lmaydigan (g) qonuniyatlar

1) *chiziqli funksiya*: $y = kx+b$

2) *kvadrat funksiya*: $y = ax^2+bx+c$

3) *darajali funksiya*: $y = x^n$

4) *kasr darajali funksiya*: $y = \sqrt[n]{x^m}$

5) *teskari proporsionallik funksiyasi*: $y = \dfrac{k}{x}$ (bu yerda $k \neq 0$)

6) *modulli funksiya*: $y = |x|$

Funksiyaning berilish usullari

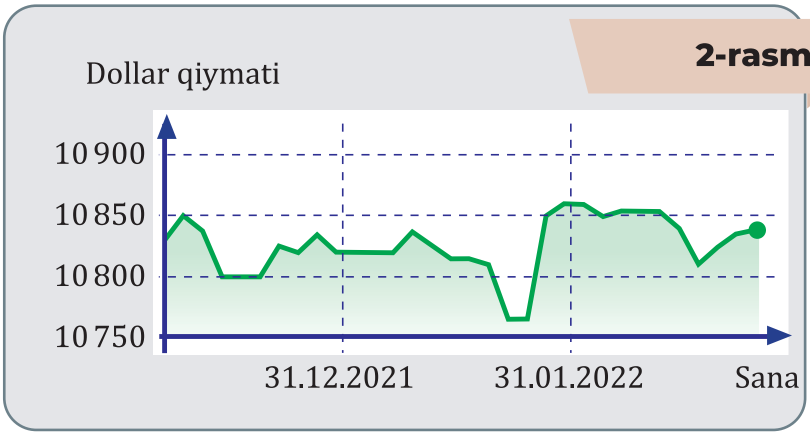
Funksiyalar quyidagi usullarda berilishi mumkin:

1. Funksiya berilishining ***analitik usuli.*** Agar funksiya bitta yoki bir nechta formula yoki tenglamalar bilan berilgan bo‘lsa, u holda bu funksiya analitik usulda berilgan deyiladi. Masalan, moddiy nuqtaning harakat tenglamasi $s = 20 - 5t + \frac{1}{4}t^2$ analitik usulda berilgan funksiyaga misol bo‘la oladi.

2. Funksiya berilishining ***jadval usuli*** odatda amaliy tajribalarda o‘zgaruvchilar orasidagi o‘zaro bog‘liqlikni ifodalaydi. Masalan, haroratning kunlik o‘zgarishi jadval usulida berilishi mumkin. Bu yerda kunlar erkli o‘zgaruvchi (ya’ni argument), harorat esa erksiz o‘zgaruvchi (ya’ni funksiya) bo‘ladi. Toshkent shahrida 2022-yil 20–26-yanvar kunlari havo haroratining haftalik o‘zgarishi quyidagi jadvalda keltirilgan:

Sana		20.01	21.01	22.01	23.01	24.01	25.01	26.01
Harorat, t °C	Kunduzi	13	9	3	4	6	7	8
	Kechasi	–2	–3	–1	–2	–3	–4	–3

3. Ayrim amaliy ishlarda o‘zgaruvchilarning bog‘liqligi ***grafik usulda*** beriladi. Masalan, dollarning so‘mga nisbatan qiymatining oylik, yillik o‘zgarishi grafik usulda ifodalanishi mumkin. Bu yerda sanalar *argument*, dollarning so‘mga nisbatan qiymati esa *funksiya* bo‘ladi.



2-rasm

4. Funksiya ***matn usulida*** berilishi ham mumkin. Masalan: 4 nafar a’zosi bor oila osh pishirish uchun 1 kg guruch sarflaydi. Uyga 2 nafar mehmon kelganda osh pishirish uchun necha kg guruch kerak bo‘ladi? Bu masalada oshdagi guruch miqdori uydagi kishilar soniga bog‘liq bo‘lib, kishilar soni argument, guruch miqdori funksiya bo‘ladi.

MISOLLAR

1. Matn bilan berilgan funksiyaning analitik ko‘rinishini yozing. Masalan, “argumentning kvadratidan 5 ni ayiring” deyilishi quyidagi funksiyani beradi: $f(x) = x^2 - 5$.

a) argumentni 3 ga ko‘paytirib, undan 5 ni ayiring.
b) argumentning kvadratiga 2 ni qo‘shing.
c) argumentdan 1 ni ayirib, keyin kvadratga ko‘taring.
d) argumentga 1 ni qo‘shing, keyin kvadrat ildizini topib, 6 ga bo‘ling.

2. Matn usulida berilgan funksiyaning ***analitik***, ***jadval*** va ***grafik*** ko‘rinishini toping:

a) $f(x)$ ni topish uchun argumentni 3 ga bo‘ling, keyin $\frac{2}{3}$ ni qo‘shing.

b) $g(x)$ ni topish uchun argumentdan 4 ni ayiring, keyin $\frac{3}{4}$ ga ko‘paytiring.

c) $T(x)$ funksiya x so‘mga sotib olingan mahsulotning soliq miqdori funksiyasi bo‘lsin, soliq miqdorini topish uchun mahsulot narxining 8 % ini hisoblang.

d) $V(d)$ funksiya d diametrli sharning hajmini topish funksiyasi bo‘lsin, hajmni topish uchun diametrning 3-darajasini π ga ko‘paytirib 6 ga bo‘ling.

3. Berilgan funksiyalar uchun qiymatlar jadvalini to'ldiring:

a) $f(x) = 2(x-1)^2$

x	$f(x)$
–1	
0	
1	
2	
3	

b) $g(x) = |2x+3|$.

x	$g(x)$
-3	
-2	
0	
1	
3	

4. Funksiyaning berilgan argumentdagi qiymatlarini toping.

a) $f(x) = x^2-6$ $\quad f(-3),\ f(3),\ f(0),\ f\left(\frac{1}{2}\right)$

b) $f(x) = x^3+2x$ $\quad f(-2),\ f(-1),\ f(0),\ f\left(\frac{1}{2}\right)$

c) $f(x)=\frac{|x|}{x}$ $\quad f(-2), f(-1), f(0), f(5), f(x^2), f\left(\frac{1}{x}\right)$

d) $f(x)=\frac{1-2x}{3}$ $\quad f(2), f(-2), f\left(\frac{1}{2}\right), f(a), f(-a), f(a-1)$

e) $h(x)=\frac{x^2+4}{5}$ $\quad h(2), h(-2), h(a), h(-x), h(a-2), h(\sqrt{x})$

f) $f(x)=x^2+2x$ $\quad f(0), f(3), f(-3), f(a), f(-x), f\left(\frac{1}{a}\right)$

g) $h(t)=t+\frac{1}{t}$ $\quad h(-1),\ h(2),\ h\left(\frac{1}{2}\right),\ h(x-1),\ h\left(\frac{1}{x}\right)$

5. Berilgan tengliklardan qaysilari x o'zgaruvchili funksiya bo'la oladi?

a) $3x-5y=7$ b) $3x^2-y=5$ c) $x=y^2$ d) $x^2+(y-1)^2=4$

e) $2x-4y^2=3$ f) $2x^2-4y^2=3$ g) $2xy-5y^2=4$ h) $\sqrt{y}-x=5$

i) $2|x|+y=0$ j) $2x+|y|=0$ k) $x=y^3$ l) $x=y^4$

6. Quyidagi jadvallardan qaysi biri x o'zgaruvchili funksiya bo'la oladi?

a)

x	y
–5	–12
9	2
11	2

b)

x	y
–10	–9
$3\frac{1}{2}$	–6
–10	–1

c)

x	y
2	0
–5	–3
–17	7
6	17
11	7

d)

x	y
–4	$3\frac{1}{2}$
$3\frac{1}{2}$	$-3\frac{1}{2}$
$9\frac{3}{5}$	–10

FUNKSIYANING ANIQLANISH SOHASI VA QIYMATLAR TO'PLAMI

Funksiyaning aniqlanish sohasi va qiymatlar to'plami

$y = f(x)$ funksiyada x argument qabul qilishi mumkin bo'lgan sonlar to'plami berilgan funksiyaning ***aniqlanish sohasi***, y funksiya qabul qilishi mumkin bo'lgan sonlar to'plami berilgan funksiyaning ***qiymatlar to'plami*** deb ataladi va ular mos ravishda $D(f)$ va $E(f)$ yoki $D(y)$ va $E(y)$ kabi belgilanadi.

Ba'zi funksiyalar uchun aniqlanish sohasi va qiymatlar to'plami jadvali:

Funksiya	Aniqlanish sohasi	Qiymatlar to'plami
1) $y=kx+b$	$D(y)=(-\infty;+\infty)$	$E(y)=(-\infty;\ +\infty)$
2) $y=x^2$	$D(y)=(-\infty;+\infty)$	$E(y)=[0;\ +\infty)$
3) $y=\|x\|$	$D(y)=(-\infty;+\infty)$	$E(y)=[0;\ +\infty)$
4) $y=\frac{k}{x}$	$D(y)=(-\infty;0)\cup(0;+\infty)$	$E(y)=(-\infty;0)\cup(0;+\infty)$
5) $y=\sqrt{x}$	$D(y)=[0;+\infty)$	$E(y)=[0;\ +\infty)$
6) $y=\sqrt[2n]{x}$	$D(y)=[0;+\infty)$	$E(y)=[0;\ +\infty)$
7) $y=\sqrt[3]{x}$	$D(y)=(-\infty;+\infty)$	$E(y)=(-\infty;\ +\infty)$
8) $y=\sqrt[2n+1]{x}$	$D(y)=(-\infty;+\infty)$	$E(y)=(-\infty;\ +\infty)$

x argumentning $y = f(x)$ funksiyaning aniqlanish sohasiga tegishli bo'lmagan har qanday qiymatida $y = f(x)$ funksiya aniqlanmagan bo'ladi, boshqacha aytganda, $f(x)$ funksiya ma'noga ega bo'lmaydi. Masalan, $y=\sqrt{x}$ funksiya $x = -1$ bo'lganda; $y=\frac{k}{x}$ funksiya $x = 0$ bo'lganda ma'noga ega emas.

1-misol. $y=\frac{1}{x^2-x}$ funksiyaning aniqlanish sohasini toping.

Yechish. Ratsional ifodaning maxraji nolga teng bo'lishi mumkin emas, ya'ni:

$$x^2-x\neq 0$$
$$x(x-1)\neq 0$$
$$x\neq 0 \text{ va } x\neq 1.$$

Demak, x argument 0 va 1 qiymatlarni qabul qila olmaydi. Shuning uchun berilgan funksiyaning aniqlanish sohasi $D(y)=(-\infty;0)\cup(0;1)\cup(1;+\infty)$.

Javob: $D(y)=(-\infty;0)\cup(0;1)\cup(1;+\infty)$.

2-misol. $y=\sqrt{9-x^2}$ funksiyaning aniqlanish sohasini toping.

Yechish. Kvadrat ildiz ostidagi ifoda manfiy bo'la olmaydi. Ya'ni:

$$9-x^2\geq 0$$

$$(3-x)(3+x)\geq 0$$

$$-3\leq x\leq 3.$$

Demak, x argument faqat [–3; 3] kesmadan qiymat qabul qila oladi. Shuning uchun funksiyaning aniqlanish sohasi: $D(y)=[-3;3]$.

Javob: $D(y)=[-3;3]$.

3-misol. $y=\frac{1}{\sqrt{x+1}}$ funksiyaning aniqlanish sohasini toping.

Yechish. Berilgan funksiya maxrajida kvadrat ildiz ostidagi ifoda berilgan, bu ifoda nolga teng bo'lishi mumkin emas hamda manfiy bo'lmasligi kerak. Shuning uchun

$$x+1>0$$

$$x>-1.$$

Demak, funksiyaning aniqlanish sohasi $D(y)=(-1;\infty)$.

Javob: $D(y)=(-1;\infty)$.

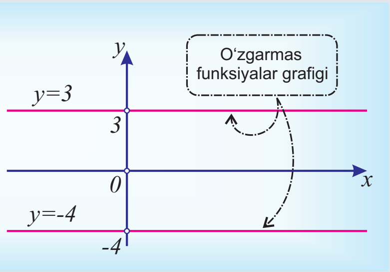
Funksiya grafigi

$y=f(x)$ funksiya o'zining $D(f)$ aniqlanish sohasidan olingan har bir x elementga $E(f)$ qiymatlar to'plamidan yagona $f(x)$ qiymatni mos qo'yadi. Natijada har bir $x\in D(f)$ element Oxy koordinatalar tekisligida yagona $(x, f(x))$ nuqtani aniqlaydi.

Oxy koordinatalar tekisligida hosil qilingan barcha $(x, f(x))$ nuqtalar to'plami $y=f(x)$ ***funksiyaning grafigi*** deyiladi.

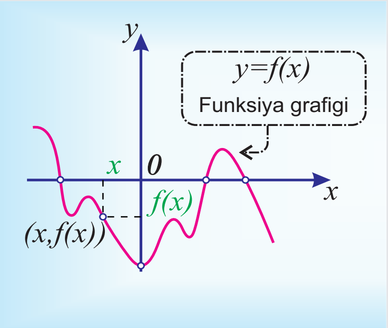
1-, 2-rasmlarda funksiya grafiklari tasvirlangan.

1-rasm



Funksiya grafigi

2-rasm



Funksiya grafigi

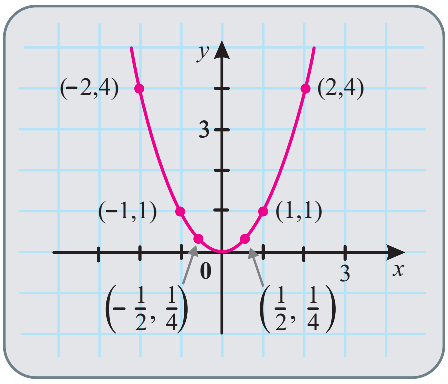
4-misol. Quyidagi funksiyalarning grafiklarini yasang.

a) $y=x^2$ b) $y=x^3$ d) $y=\sqrt{x}$

Yechish. Bu funksiyalarning grafiklarini yasash uchun dastlab ayrim qiymatlar uchun jadvalini tuzib olamiz. Keyin bu nuqtalarni koordinata tekisligida belgilaymiz va ularni egri chiziq bilan tutashtiramiz.

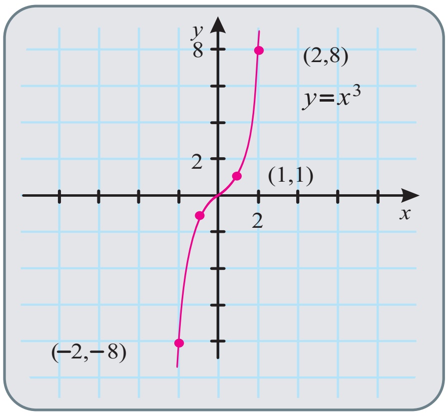
a)

x	–3	–2	–1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9



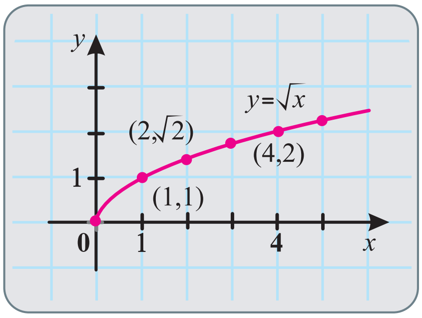
b)

x	–2	–1	–1/2	0	1/2	1	2
$y = x^3$	–8	–1	–1/8	0	1/8	1	8



c)

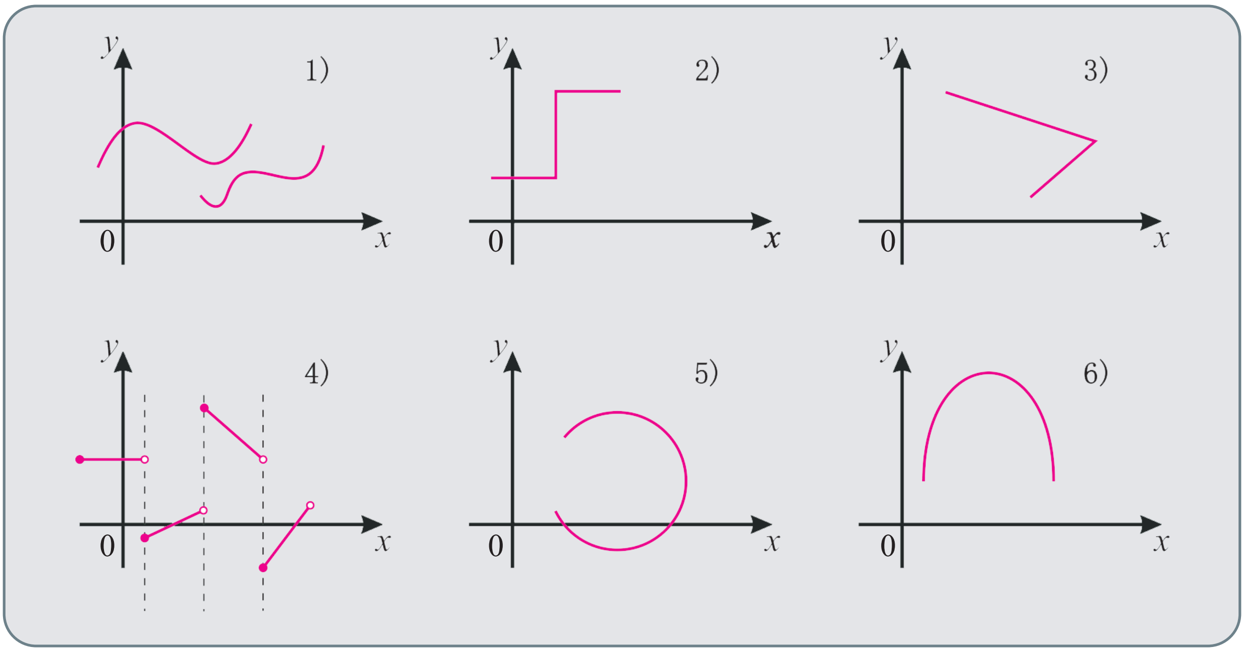
x	0	1/4	1	2	4	9
$y=\sqrt{x}$	0	1/2	1	$\sqrt{2}$	2	3



Oy o'qiga parallel ixtiyoriy to'g'ri chiziq berilgan grafikni bittadan ortiq bo'lmagan nuqtada kesib o'tsa Oxy tekisligida tasvirlangan shakl $y = f(x)$ funksiyaning grafigi bo'ladi.

Agar Oy o'qiga parallel qandaydir to'g'ri chiziq berilgan chiziqni bittadan ortiq nuqtada kesib o'tgan taqdirda Oxy tekisligida tasvirlangan chiziq funksiya grafigi bo'la olmaydi.

Quyidagi rasmda keltirilgan 4- va 6-chiziqlar biror funksiya ning grafigi bo'ladi. 1-, 2-, 3- va 5-chiziqlar esa funksiya grafigi bo'lmaydi.



MISOLLAR

1. Funksiyaning aniqlanish sohasini va qiymatlar to'plamini toping.

a) $f(x)=3x$ b) $f(x)=3x,\ 2\le x\le 6$

c) $f(x)=5x^2+2$ d) $f(x)=5x^2+2,\ 0\le x\le 2$

2. Funksiyaning aniqlanish sohasini toping.

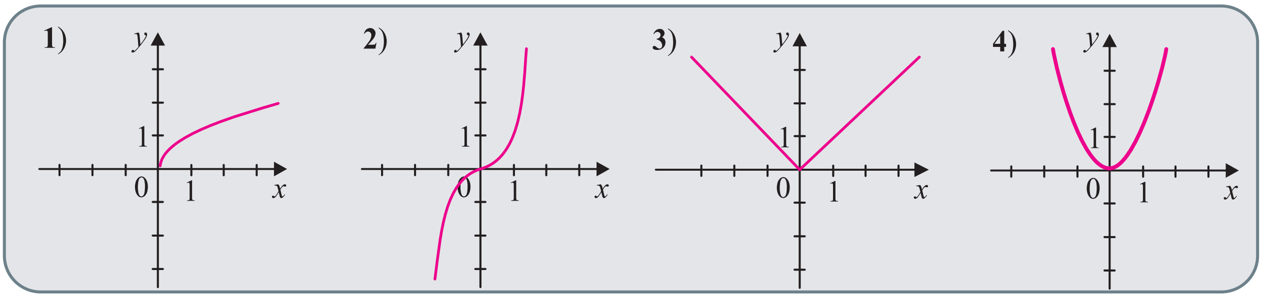
a) $f(x)=\dfrac{1}{x-3}$ b) $f(x)=\dfrac{1}{3x-6}$ c) $f(x)=\dfrac{x+2}{x^2-1}$

d) $f(x)=\dfrac{x^4}{x^2+x-6}$ e) $f(t)=\sqrt{t+1}$ f) $g(t)=\sqrt{t^2+9}$

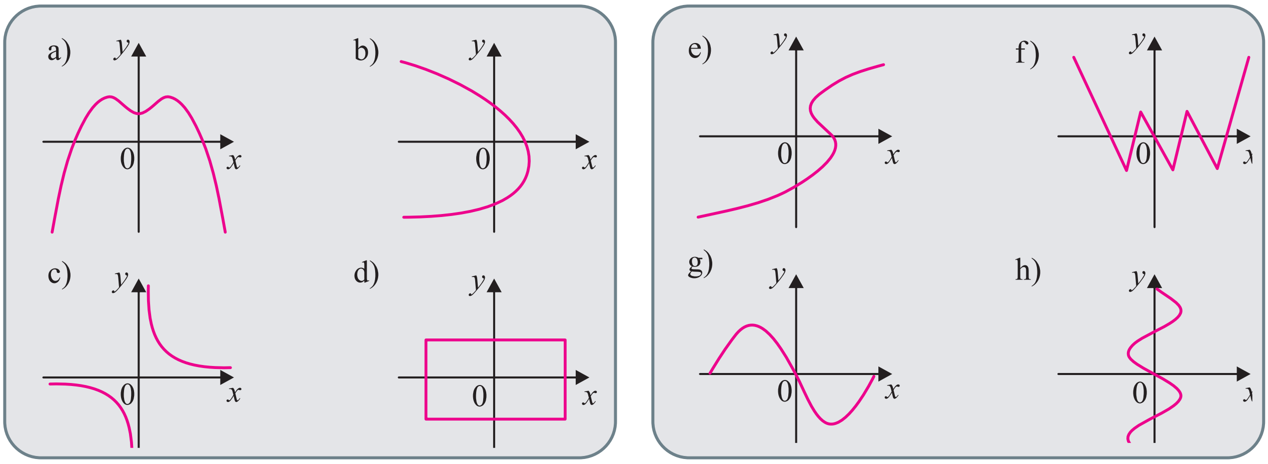
g) $f(t)=\sqrt[3]{t-1}$ h) $g(x)=\sqrt{7-3x}$ i) $f(x)=\sqrt{1-2x}$

3. Funksiyaga mos grafikni aniqlang.

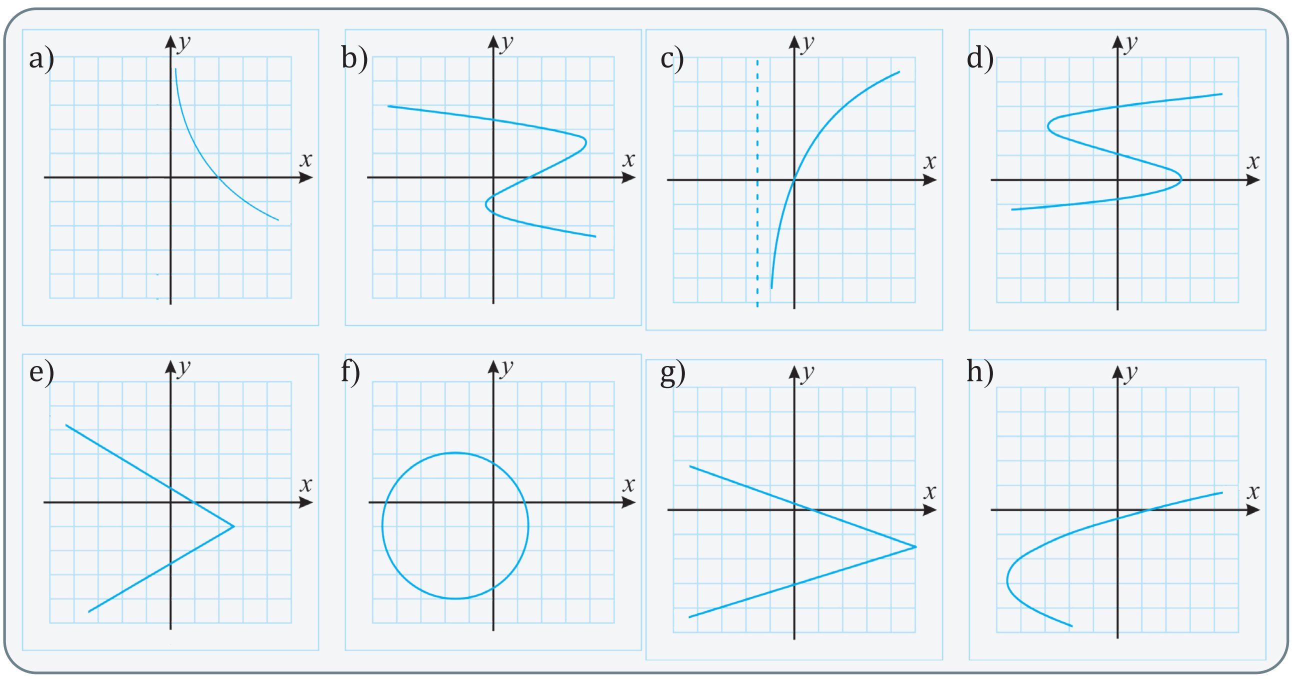
a) $f(x)=x^2$ b) $f(x)=x^3$ c) $f(x)=\sqrt{x}$ d) $f(x)=|x|$



4. Berilganlardan qaysilari funksiya grafigi bo'la oladi?



5. Berilganlardan qaysilari funksiya grafigi bo'la olmaydi?



6. Berilgan funksiyalarning grafiklarini yasang.

a) $f(x)=8x-x^2$ b) $g(x)=x^2-x-20$ c) $h(x)=x^3-5x-4$

7. Berilgan funksiyalarning qiymatlar jadvalini tuzing va grafigini yasang.

a) $f(x)=-x^2$ b) $f(x)=x^2-4$ c) $g(x)=-(x+1)^2$

d) $r(x)=3x^4$ e) $r(x)=1-x^4$ f) $g(x)=x^3-8$

g) $k(x)=\sqrt[3]{-x}$ h) $k(x)=-\sqrt[3]{x}$ i) $f(x)=1+\sqrt{x}$

j) $C(t)=\dfrac{1}{t^2}$ k) $C(t)=-\dfrac{1}{t+1}$ l) $H(x)=|2x|$

m) $G(x)=|x|+x$ n) $G(x)=|x|-x$ o) $f(x)=|2x-2|$

FUNKSIYALAR USTIDA ARIFMETIK AMALLAR

Funksiyalar ustida arifmetik amallar

Funksiyalar ustida qo'shish (+), ayirish (–), ko'paytirish (×), bo'lish (÷) arifmetik amallarini bajarish mumkin.

$f(x)$ va $g(x)$ funksiyalarning aniqlanish sohalari mos ravishda A va B to'plamlar bo'lsin. Bu funksiyalarning $A \cap B$ to'plamdagi ***yig'indisi*** deb har bir $x \in A \cap B$ elementda $f(x) + g(x)$ qiymatni qabul qiladigan funksiyaga aytiladi. $f(x)$ va $g(x)$ funksiyalarning yig'indisi $(f + g)(x)$ kabi belgilanadi. Demak:

$$(f + g)(x) = f(x) + g(x).$$

Xuddi shuningdek, $f(x)$ va $g(x)$ funksiyalarning ***ayirmasi, ko'paytmasi, bo'linmasini*** aniqlash mumkin:

$$(f - g)(x) = f(x) - g(x)$$
$$(f \cdot g)(x) = f(x)g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}.$$

Diqqat qiling!

1. $A \cap B = \varnothing$ bo'lsa, bu amallar aniqlanmaydi.

2. Ikkita $f(x)$ va $g(x)$ funksiyalarning bo'linmasini aniqlashda X to'plamdan olingan har bir x element uchun $g(x) \neq 0$ bo'lishi talab etiladi.

MISOLLAR

1-misol. $f(x) = \frac{1}{x-2}$ va $g(x) = \sqrt{x}$ funksiyalar berilgan.

a) $(f+g)(x), (f-g)(x), (f \cdot g)(x)$ va $\left(\frac{f}{g}\right)(x)$ funksiyalarni va ularning aniqlanish sohasini toping.

b) $(f+g)(4), (f-g)(4), (fg)(4)$ va $\left(\frac{f}{g}\right)(4)$ ni toping.

Yechish. a) f funksiyaning aniqlanish sohasi $x \neq 2$, g funksiyaniki esa $x \geq 0$. f va g funksiyalarning aniqlanish sohalari kesishmasi $[0; 2) \cup (2; \infty)$ bo'ladi.

U holda ular ustida amallar quyidagicha bajariladi:

$$(f+g)(x) = f(x) + g(x) = \frac{1}{x-2} + \sqrt{x}$$

$$(f-g)(x) = f(x) - g(x) = \frac{1}{x-2} - \sqrt{x}$$

$$(f\cdot g)(x) = f(x)g(x) = \frac{\sqrt{x}}{x-2}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{1}{(x-2)\sqrt{x}}.$$

b) $x = 4$ qiymat har bir yangi funksiyaning aniqlanish sohasiga tegishli ekanidan quyidagi qiymatlar aniqlangan:

$$(f+g)(4) = f(4) + g(4) = \frac{1}{4-2} + \sqrt{4} = \frac{5}{2}$$

$$(f-g)(4) = f(4) - g(4) = \frac{1}{4-2} - \sqrt{4} = -\frac{3}{2}$$

$$(fg)(4) = f(4)g(4) = \left(\frac{1}{4-2}\right)\sqrt{4} = 1$$

$$\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{1}{(4-2)\sqrt{4}} = \frac{1}{4}.$$

2-misol. Funksiyalarni grafik usulda qo'shish

f va g funksiyalarning grafigi 1-rasmda berilgan bo'lsin. Grafik usuldagi qo'shish yordamida $f+g$ funksiyaning grafigini chizing.

Yechish. Ma'lumki, f funksiya grafigi Oxy tekislikdagi

$$\{(x, f(x)) : x \in D(f)\}$$

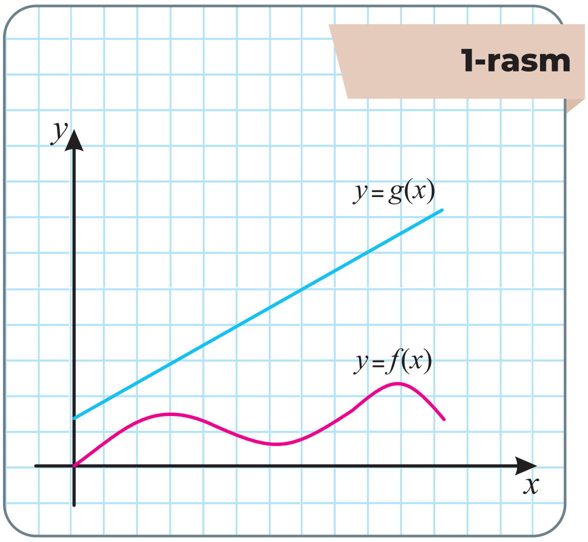
to'plamdan iborat. Xuddi shuningdek,

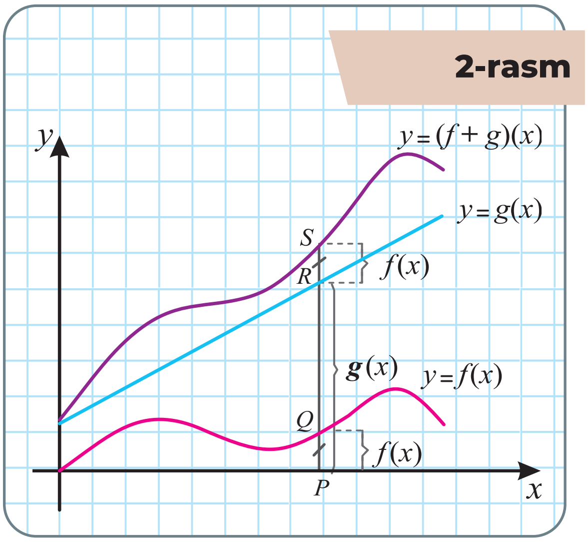
$$\{(x, g(x)) : x \in D(g)\}$$

to'plam g funksiyaning grafigi bo'ladi. f va g funksiyalarni qo'shishning grafik usuli deganda ushbu to'plam tushuniladi:

$$\{(x, f(x) + g(x)) : x \in D(f) \cup D(g)\}.$$

PQ kesma PR kesmaning yuqorisiga $f+g$ funksiyaning S nuqtasini hosil qilish uchun nusxalab ko'chirilgan ($PQ = RS$).





MISOLLAR

1. Funksiyalarni qo'shing va ayiring.

a) $f(x) = 5x + 1$, $g(x) = -2x$

b) $f(x) = -3x + 3$, $g(x) = -5x + 4$

c) $f(x) = 2x + 1$, $g(x) = -5x + 3$

d) $f(x) = -3x^2 + 7x$, $g(x) = 2x + 4$

2. Funksiyalarni ko'paytiring.

a) $f(x)=-x^2$, $g(x)=-3x+1$

b) $f(x)=-3x^2+3$, $g(x)=-x$

c) $f(x)=-x+3$, $g(x)=5x+6$

d) $f(x)=-4x+5$, $g(x)=-3x+1$

3. $(f+g)(x), (f-g)(x), (f\cdot g)(x)$ va $\left(\frac{f}{g}\right)(x)$ funksiyalarni va ularning aniqlanish sohasini toping.

a) $f(x)=x,\ g(x)=2x$

b) $f(x)=x,\ g(x)=\sqrt{x}$

c) $f(x)=x^2+x,\ g(x)=x^2$

d) $f(x)=3-x^2,\ g(x)=x^2-4$

e) $f(x)=5-x,\ g(x)=x^2-3x$

f) $f(x)=x^2+2x,\ g(x)=3x^2-1$

g) $f(x)=\sqrt{25-x^2},\ g(x)=\sqrt{x+3}$

h) $f(x)=\sqrt{16-x^2},\ g(x)=\sqrt{x^2-1}$

i) $f(x)=\frac{2}{x},\ g(x)=\frac{4}{x+4}$

j) $f(x)=\frac{2}{x+1},\ g(x)=\frac{x}{x+1}$

4. Funksiyalarning aniqlanish sohasini toping.

a) $f(x)=\sqrt{x}+\sqrt{3-x}$

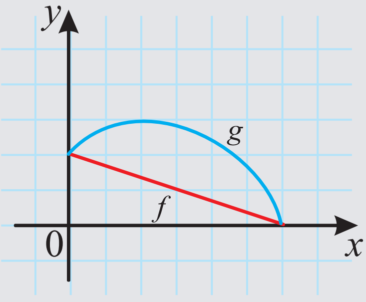
b) $f(x)=\sqrt{x+4}-\frac{\sqrt{1-x}}{x}$

c) $h(x)=(x-3)^{-\frac{1}{4}}$

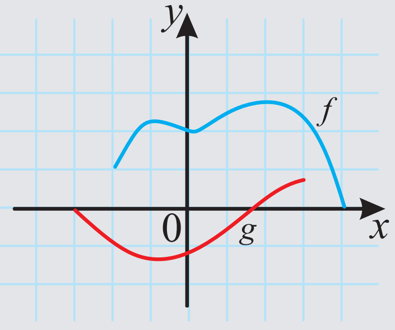
d) $k(x)=\frac{\sqrt{x+3}}{x-1}$

5. Grafikdan foydalanib qo'shish yordamida $f+g$ funksiyaning grafigini yasang (3–4-*rasmlar*).

3-rasm



4-rasm



MURAKKAB, TESKARI, DAVRIY FUNKSIYALAR

Murakkab funksiya

Funksiyalarni ketma-ket qo'llash natijasida o'zgaruvchilarning yangi bog'lanishlari hosil bo'ladi. Agar X to'plamda $y = f(x)$ funksiya berilgan bo'lib, x argument T to'plamda aniqlangan biror $x = g(t)$ funksiya bo'lsa, u holda T to'plamda $y = f(g(t))$ ***murakkab funksiya*** aniqlangan deyiladi.

Masalan, $y = 2x^2-3x$ funksiya $X = (-\infty; +\infty)$ to'plamda, $x=\sqrt{t}$ funksiya esa $T=[0;+\infty)$ to'plamda berilgan bo'lsin. U holda $y=2t-3\sqrt{t}$ funksiya $T=[0;+\infty)$ to'plamda $y=2x^2-3x$ va $x=\sqrt{t}$ funksiyalarning murakkab funksiyasi bo'ladi.

1-misol. $f(x)=x^2$ va $g(x)=x-3$ funksiyalar berilgan:

a) $f(g(x))$ va $g(f(x))$ murakkab funksiyalarni va ularning aniqlanish sohasini toping;

b) $f(g(5))$ va $g(f(7))$ ni toping.

Yechish. a) Quyidagi tengliklar o'rinli:

g funksiyaning berilishiga ko'ra, $f(g(x)) = f(x-3)$,

f funksiyaning berilishiga ko'ra, $f(g(x)) = (x-3)^2$ bo'ladi.

f funksiyaning berilishiga ko'ra, $g(f(x)) = g(x^2)$,

g funksiyaning berilishiga ko'ra, $g(f(x)) = x^2-3$ bo'ladi.

$f(g(x))$ va $g(f(x))$ funksiyaning aniqlanish sohasi haqiqiy sonlar to'plami.

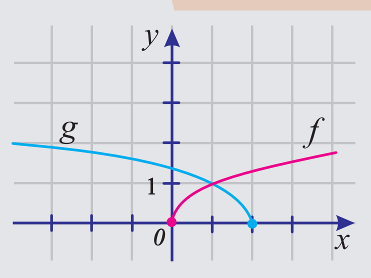
b) Topilgan murakkab funksiyalarda x ning o'rniga berilgan qiymatni qo'yamiz:

$f(g(5)) = (5-3)^2 = 2^2 = 4,\qquad g(f(7)) = 7^2-3 = 49-3 = 46.$

2-misol. Agar $f(x)=\sqrt{x}$ va $g(x)=\sqrt{2-x}$ berilgan bo'lsa, quyidagi funksiyalarni va ularning aniqlanish sohasini toping (1-*rasm*).

a) $f(g(x))$ b) $g(f(x))$

c) $f(f(x))$ d) $g(g(x))$



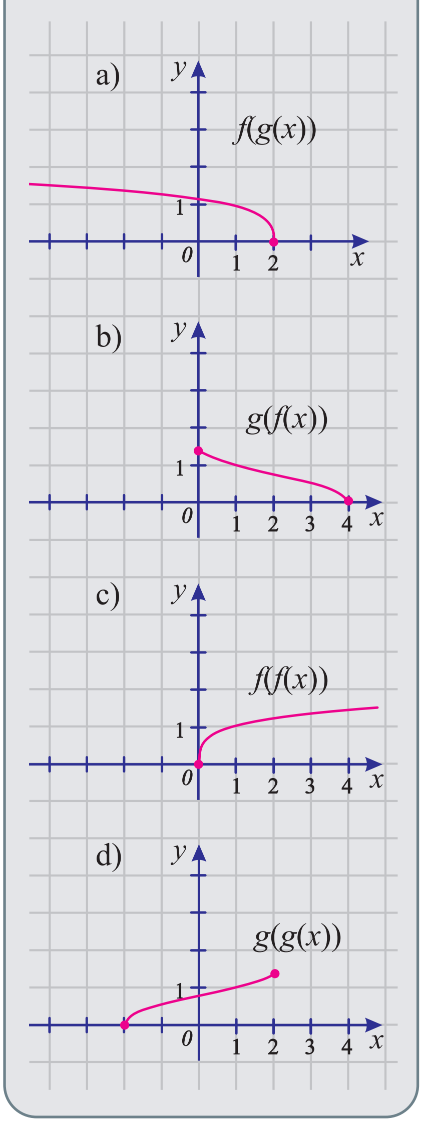
1-rasm

Yechish. a) Murakkab funksiya ta'rifi hamda f va g funksiyalarning berilishiga ko'ra,

$$f(g(x)) = f\left(\sqrt{2-x}\right)=\sqrt{\sqrt{2-x}}=\sqrt[4]{2-x} \text{ bo'ladi.}$$

$\sqrt[4]{2-x}$ ifodaning aniqlanish sohasi $2-x\geq 0$. Bundan $x \leq 2$.

2-rasm



Demak, $f(g(x))$ ning aniqlanish sohasi $(-\infty; 2]$ oraliqdan iborat (2a-*rasm*).

b) f ning berilishiga ko'ra, $g(f(x)) = g\left(\sqrt{x}\right)$ bo'lib,

g ning berilishiga ko'ra, $g(f(x)) = \sqrt{2-\sqrt{x}}$ bo'ladi.

$\sqrt{x}$ ning aniqlanish sohasi: $x \geq 0$.

$\sqrt{2-\sqrt{x}}$ ning aniqlanish sohasi: $2-\sqrt{x} \geq 0$, bundan

$\sqrt{x} \leq 2$ yoki $x \leq 4$. Demak, $0 \leq x \leq 4$ (2b-*rasm*).

c) f ning berilishiga ko'ra, $f(f(x)) = f\left(\sqrt{x}\right)$ bo'lib,

f ning berilishiga ko'ra, $f(f(x)) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$ bo'ladi.

$\sqrt[4]{x}$ ning aniqlanish sohasi: $[0; \infty)$ (2c-*rasm*).

d) g ning berilishiga ko'ra, $g(g(x)) = g\left(\sqrt{2-x}\right)$ bo'lib,

g ning berilishiga ko'ra, $g(g(x)) = \sqrt{2-\sqrt{2-x}}$ bo'ladi.

$\sqrt{2-\sqrt{2-x}}$ ning aniqlanish sohasi: $2-x \geq 0$ va $\sqrt{2-x} \leq 2$

Bundan $x \leq 2$ va $x \geq -2 \Rightarrow -2 \leq x \leq 2$, demak, $g(g(x))$ ning aniqlanish sohasi: $[-2; 2]$ (2d-*rasm*).

3-misol. $f(x) = \dfrac{x}{x+1}$, $g(x) = x^{10}$ va $h(x) = x+3$ bo'lsa, $f(g(h(x)))$ ni toping.

Yechish. h ning berilishiga ko'ra,

$f(g(h(x))) = f\left(g(x+3)\right)$ bo'lib, g funksiyaning berilishiga ko'ra, $f(g(h(x))) = f\left((x+3)^{10}\right)$,

f funksiyaning berilishiga ko'ra, $f(g(h(x))) = \dfrac{(x+3)^{10}}{(x+3)^{10}+1}$ bo'ladi.

4-misol. $F(x) = \sqrt[4]{x+9}$ funksiya berilgan. $F(x) = f(g(x))$ bo'ladigan f va g funksiyalarga misol keltiring.

Yechish. f va g funksiyalarni quyidagicha olishimiz mumkin: $g(x) = x+9$ va $f(x) = \sqrt[4]{x}$.

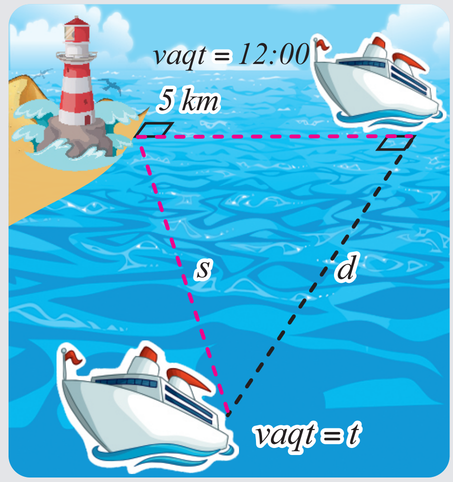
Bunda, g ning berilishiga ko'ra, $f(g(x)) = f(x+9)$ bo'lib, f ning berilishiga ko'ra, $f(g(x)) = \sqrt[4]{x+9}$ bo'ladi.

Ushbu topshiriq shartini qanoatlantiruvchi f va g funksiyalarni bir nechta holatda tanlab olish mumkin. Shulardan yana biri $f(x)=\sqrt{x}$ va $g(x)=\sqrt{x+9}$.

5-misol. Murakkab funksiyaning qo'llanishi

Kema 20 km/h o'zgarmas tezlikda qirg'oqqa parallel ravishda harakat qilmoqda. Kema mayoqning ro'parasidan soat 12:00 da, qirg'oqdan 5 km uzoqlikda o'tadi.

3-rasm



a) Mayoq va kema orasidagi s masofani kemaning soat 12:00 dan keyin yurgan masofasi d ga bog'liq funksiyasi ko'rinishida yozing:

$$s=f(d).$$

b) d ni soat 12:00 dan keyin o'tgan vaqt t (soat) ga bog'liq funksiyasi ko'rinishida yozing:

$$d=g(t).$$

c) $f(g(t))$ murakkab funksiyani toping. Bu funksiya nimani anglatadi?

Yechish. 3-rasmga qaraymiz.

a) s va d masofalarning bog'liqligini Pifagor teoremasi yordamida ko'rsatamiz. Boshqacha aytganda, s ning d ga bog'liq funksiya ekanini quyidagicha ifodalaymiz:

$$s=f(d)=\sqrt{25+d^2}.$$

b) kema 20 km/h o'zgarmas tezlikda harakatlanayotgani uchun d masofaning t vaqtga bog'liqligi

$$d=g(t)=20t$$

ko'rinishdagi funksiya orqali ifodalanishi mumkin.

c) shunday qilib:

g ning berilishiga ko'ra, $f(g(t))=f(20t)$ bo'lib,

f funksiyaning berilishiga ko'ra, $f(g(t))=\sqrt{25+(20t)^2}$ bo'ladi. Bu yerda $f(g(t))$ funksiya kema va mayoq orasidagi masofaning vaqtga nisbatan funksiyasini anglatadi.

Teskari funksiya

Agar $f(x)=y$ tenglama har bir y uchun x ga nisbatan yagona $g(y)$ ildizga ega bo'lsa, u holda $x=g(y)$ funksiya $y=f(x)$ funksiyaga ***teskari funksiya*** deyiladi. $x=g(y)$ funksiya o'rniga odatdagi belgilashlarga ko'ra, $y=g(x)$ yozuvi ishlatiladi. $y=f(x)$ funksiyaga teskari funksiya $y=f^{-1}(x)$ kabi yoziladi.

1-misol. $y=3x-5$ funksiyani ko'rib chiqaylik. Bu yerdan x ni y orqali ifodalaylik:

$$3x-5=y \Rightarrow 3x=y+5 \Rightarrow x=\frac{y+5}{3}.$$

Oxirgi tenglikda x va y larning o'rinlarini almashtirib:

$$y=\frac{x+5}{3}$$

funksiyaga ega bo'lamiz. Demak, $f^{-1}(x)=\frac{x+5}{3}$ funksiya $y=3x-5$ funksiyaga teskari funksiya bo'ladi.

Eslatma. Berilgan $y=f(x)$ funksiya va unga teskari $y=f^{-1}(x)$ funksiya uchun $D\left(f^{-1}\right)=E(f)$ hamda $E\left(f^{-1}\right)=D(f)$ bo'ladi.

Diqqat qiling! $\left(f(x)\right)^{-1}=\frac{1}{f(x)}$ bo'lib, bu tenglikdagi (-1) daraja ko'rsatkichini anglatadi. $f^{-1}(x)$ yozuvdagi (-1) esa teskari funksiyani bildiradi. Umuman olganda, $\left(f(x)\right)^{-1}\neq f^{-1}(x)$. Masalan:

$f(x)=3x-5$ funksiya uchun $f^{-1}(x)=\frac{x+5}{3}$ hamda $\left(f(x)\right)^{-1}=\frac{1}{3x-5}$ bo'ladi.

2-misol. Berilgan funksiyaning teskari funksiyasini toping: $f(x)=\frac{x^5-3}{2}$.

Yechish. Funksiyani $y=\frac{x^5-3}{2}$ kabi yozib olamiz va x ni y orqali ifodalaymiz:

$$y=\frac{x^5-3}{2}$$

$$2y=x^5-3$$

$$x^5=2y+3$$

$$x=\sqrt[5]{2y+3}.$$

Endi x va y larning o'rnini almashtiramiz: $y=\sqrt[5]{2x+3}$. Demak, teskari funksiya quyidagicha: $f^{-1}(x)=\sqrt[5]{2x+3}$.

3-misol. Berilgan funksiyaning teskari funksiyasini toping: $f(x)=\frac{2x+3}{x-1}$.

Yechish. Funksiyani quyidagicha yozib olamiz: $y=\frac{2x+3}{x-1}$, x ni esa y orqali ifodalaymiz:

$$y=\frac{2x+3}{x-1}$$

$$y\cdot(x-1)=2x+3$$

$$yx-y=2x+3$$

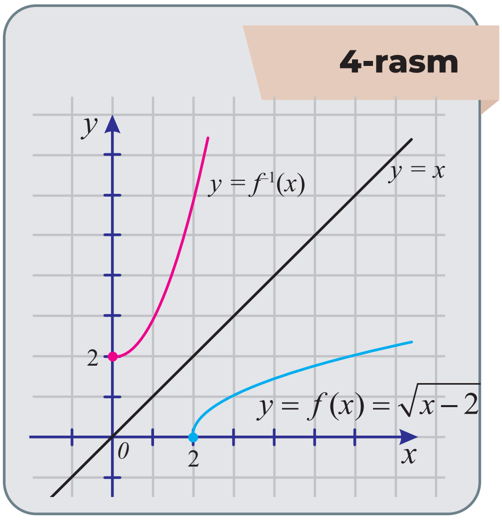
$$yx-2x=y+3$$

$$x\cdot(y-2)=y+3$$

$$x=\frac{y+3}{y-2}$$

Demak, $f^{-1}(x)=\dfrac{x+3}{x-2}$ teskari funksiya bo'ladi.

4-misol. Teskari funksiyaning grafigini yasash.



$f(x)=\sqrt{x-2}$ funksiyaning grafigidan foydalanib f^{-1} funksiyaning grafigini yasang va uning analitik ko'rinishini yozing.

Yechish.

1. $y=\sqrt{x-2}$ funksiyaning grafigi 4-rasmda keltirilgan.

2. f^{-1} funksiyaning grafigi f funksiyaning grafigini $y=x$ to'g'ri chiziqqa nisbatan simmetrik akslantirish yordamida yasaladi (4-*rasm*).

3. $y=\sqrt{x-2}$ funksiyada x ni y orqali ifodalanadi, bunda $y\ge 0$ ekani inobatga olinadi.

$$\sqrt{x-2}=y$$

$$x-2=y^2$$

$$x=y^2+2,\quad y\ge 0.$$

Endi x va y larning o'rnini almashtiramiz: $y=x^2+2$, $x\ge 0$.

Demak, teskari funksiya $f^{-1}(x)=x^2+2$ bo'lar ekan, $x\ge 0$.

Bu topilgan $f^{-1}(x)$ teskari funksiya $y=x^2+2$ parabolaning o'ng tarmog'idan iborat. Buni grafikdan ham ko'rsa bo'ladi.

Davriy funksiyalar

$D(f)$ berilgan $y=f(x)$ funksiyaning aniqlanish sohasi bo'lsin. Shunday $T\neq 0$ topilib, har bir $x\in D(f)$ uchun:

1. $x-T$ va $x+T$ lar $D(f)$ ga tegishli,

2. $f(x-T)=f(x)=f(x+T)$ munosabatlar bajarilsa, u holda $y=f(x)$ ***davriy funksiya*** deyiladi.

Agar T soni $y=f(x)$ funksiyaning davri bo'lsa, u holda har bir n butun son uchun nT soni ham $y=f(x)$ funksiyaning davri bo'ladi:

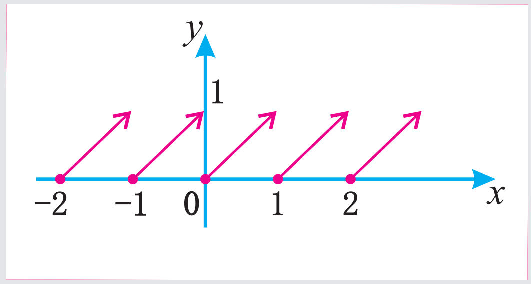
$$f(x+nT)=f(x),\quad n\in\mathbb{Z}.$$

Eng kichik musbat T davr $f(x)$ funksiyaning ***asosiy davri*** deb yuritiladi.

Davriy funksiyaning grafigini bitta davr oralig'ida chizish kifoya qiladi, boshqa davr oraliqlarida shu grafik takrorlanadi.

Masalan, sonning kasr qismi $\{x\}$ – berilgan x songa uning kasr qismini mos qo‘yuvchi funksiya (5-*rasm*) davriy funksiya bo‘ladi. Uning asosiy davri $T_0 = 1$, ya’ni ixtiyoriy $x \in (-\infty; +\infty)$ son uchun $(x + 1) \in (-\infty; +\infty)$ hamda $\{x+1\} = \{x\}$ munosabatlar o‘rinli bo‘ladi.

5-rasm



$y = \{x\}$ funksiya grafigi

Agar $y = f(x)$ funksiyaning asosiy davri T_0 bo‘lsa, u holda $y = kf(ax+b)+c$ funksiyaning asosiy davri $T_1 = \frac{T_0}{|a|}$ bo‘ladi ($a \neq 0$).

MISOLLAR

1. $f(x) = 2x - 3$ va $g(x) = 4 - x^2$ dan foydalanib murakkab funksiyalarning qiymatini toping.

a) $f(g(0))$ b) $g(f(0))$ c) $f(f(2))$ d) $g(g(3))$

e) $f(g(-2))$ f) $g(f(-2))$ g) $f(f(-1))$ h) $g(g(-1))$

2. $f(g(x))$, $g(f(x))$, $f(f(x))$ va $g(g(x))$ funksiyalarni va ularning aniqlanish sohasini toping.

a) $f(x) = 2x + 3$, $g(x) = 4x - 1$ b) $f(x) = 6x - 5$, $g(x) = \frac{x}{2}$

c) $f(x) = x^2$, $g(x) = x + 1$ d) $f(x) = x^3 + 2$, $g(x) = \sqrt[3]{x}$

e) $f(x) = \frac{1}{x}$, $g(x) = 2x + 4$ f) $f(x) = x^2$, $g(x) = \sqrt{x-3}$

g) $f(x) = |x|$, $g(x) = 2x + 3$ h) $f(x) = 4 - x$, $g(x) = |x + 4|$

i) $f(x) = \frac{x}{x+1}$, $g(x) = 2x - 1$ j) $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = x^2 - 4x$

k) $f(x) = \frac{x}{x+1}$, $g(x) = \frac{1}{x}$ l) $f(x) = \frac{2}{x}$, $g(x) = \frac{x}{x+2}$

3. $f(x) = 3 - x$ va $g(x) = x^2 + 1$ dan foydalanib funksiyalarni toping.

a) $f(g(x))$ b) $g(f(x))$ c) $f(f(x))$ d) $g(g(x))$

4. $f(g(h(x)))$ murakkab funksiyani toping.

a) $f(x) = x - 1$, $g(x) = \sqrt{x}$, $h(x) = x - 1$

b) $f(x) = \frac{1}{x}$, $g(x) = x^3$, $h(x) = x^2 + 2$

c) $f(x)=x^4+1$, $g(x)=x-5$, $h(x)=\sqrt{x}$

d) $f(x)=\sqrt{x}$, $g(x)=\dfrac{x}{x-1}$, $h(x)=\sqrt[3]{x}$

5. $F(x) = f(g(x))$ tenglikni qanoatlantiradigan f va g sodda funksiyalarga misollar keltiring.

a) $F(x)=(x-9)^5$ b) $F(x)=\sqrt{x}+1$ c) $F(x)=\dfrac{x^2}{x^2+4}$

d) $F(x)=\dfrac{1}{x+3}$ e) $F(x)=\left|1-x^3\right|$ f) $F(x)=\sqrt{1+\sqrt{x}}$

6. Berilgan f funksiyaga teskari funksiyani toping.

a) $f(x)=3x+5$ b) $f(x)=7-5x$ c) $f(x)=5-4x^3$

d) $f(x)=3x^3+8$ e) $f(x)=\dfrac{1}{x+2}$ f) $f(x)=\dfrac{5}{x-6}$

g) $f(x)=\dfrac{3-4x}{8x-1}$ h) $f(x)=\dfrac{3x}{x-2}$ i) $f(x)=\dfrac{2x+5}{x-7}$

j) $f(x)=\sqrt{5+8x}$ k) $f(x)=2+\sqrt[3]{x}$ l) $f(x)=x^6$, $x\ge 0$

m) $f(x)=\dfrac{1}{x^2}$, $x>0$ n) $f(x)=4-x^2$, $x\ge 0$ o) $f(x)=x^2+x$, $x\ge -\dfrac{1}{2}$

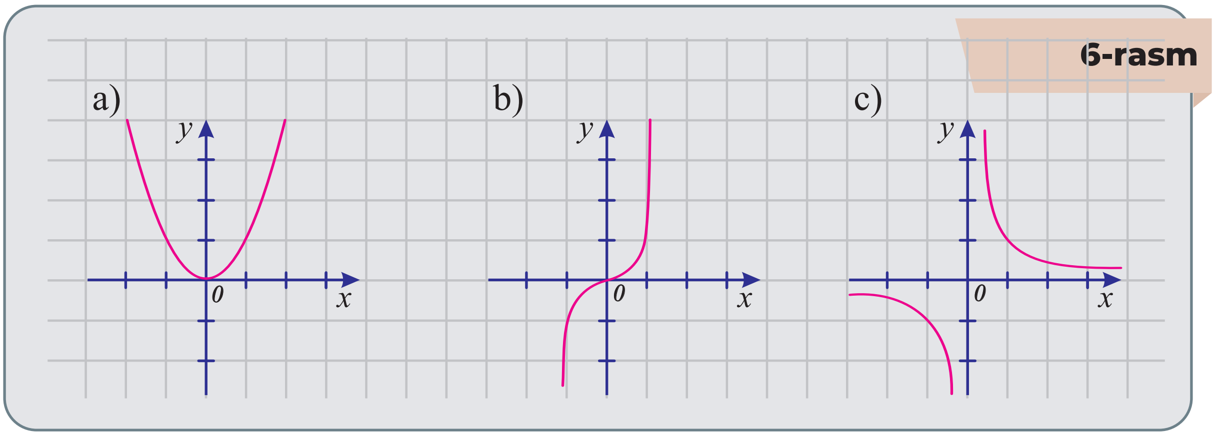
7. Berilgan funksiyaga teskari funksiyani toping. f funksiyaning grafigidan foydalanib teskari funksiya grafigini yasang.

a) $f(x)=3x-6$ b) $f(x)=16-x^2$, $x\ge 0$

c) $f(x)=\sqrt{x+1}$ d) $f(x)=x^3$

8. 6-rasmda berilgan grafiklarga mos funksiyalarni tanlang va ularga teskari funksiya grafigini yasang:

1) $f(x)=x^3$; 2) $f(x)=\dfrac{1}{x}$; 3) $f(x)=x^2$



6-rasm

9. $T=\sqrt{2}$ son $f(x)=5$ funksiyaning davri bo'lishini isbotlang.

10. Berilgan funksiyalar davriy emasligini ko'rsating.

a) $f(x)=\dfrac{1}{x-3}$ b) $f(x)=-\dfrac{2}{x-2}$ c) $f(x)=\dfrac{x}{x}$

d) $f(x)=x^2-4$ e) $f(x)=\dfrac{x^2+2}{x^2+5x+8}$ f) $f(x)=\sqrt[3]{x}+3x-1$

FUNKSIYA XOSSALARI

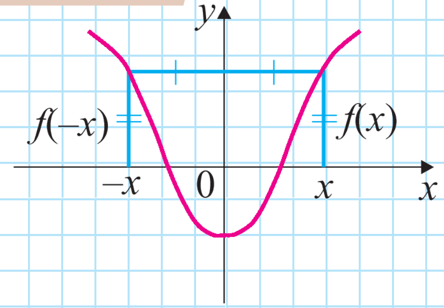
Juft va toq funksiyalar

Ixtiyoriy $x \in D(f)$ uchun $f(-x) = f(x)$ tenglik bajarilsa, u holda $f(x)$ ***juft funksiya*** deyiladi. Juft funksiyaning grafigi Oy o'qiga nisbatan simmetrik bo'ladi (1-*rasm*).

Ixtiyoriy $x \in D(f)$ uchun $f(-x) = -f(x)$ tenglik bajarilsa, u holda $f(x)$ ***toq funksiya*** deyiladi. Toq funksiyaning grafigi koordinata boshiga nisbatan simmetrik bo'ladi (2-*rasm*).

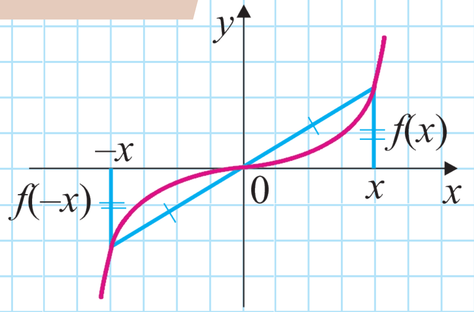
Yuqoridagi ikkita tenglikdan birortasi ham bajarilmasa, u holda $f(x)$ ***juft ham, toq ham emas funksiya*** deyiladi.

1-rasm



$f(x)$ – juft funksiya

2-rasm



$f(x)$ – toq funksiya

1-misol. $f(x) = 2x^2 + 5$ funksiyani juft yoki toqligini tekshiring.

Yechish

$f(x) = 2x^2 + 5$ funksiya uchun:

$$f(-x) = 2(-x)^2 + 5 = 2x^2 + 5 = f(x)$$

ekanidan $f(x)$ funksiya juft funksiya bo'ladi.

2-misol. $f(x) = 2x^3 + 5x$ funksiyani juft yoki toqligini tekshiring.

Yechish

$f(x) = 2x^3 + 5x$ funksiya uchun:

$$f(-x) = 2(-x)^3 + 5(-x) = -(2x^3 + 5x) = -f(x)$$

ekanligidan $f(x)$ funksiya toq funksiya bo'ladi.

3-misol. $f(x) = 2x^3 + 5x^2 - 3x + 1$ funksiyani juft yoki toqligini tekshiring.

Yechish

$$f(-x) = 2(-x)^3 + 5(-x)^2 - 3(-x) + 1 =$$

$$= -(2x^3 - 5x^2 - 3x - 1)$$

Demak, $f(-x) \neq f(x)$, $f(-x) \neq -f(x)$ bo'lib, bu funksiya juft ham, toq ham emas ekan.

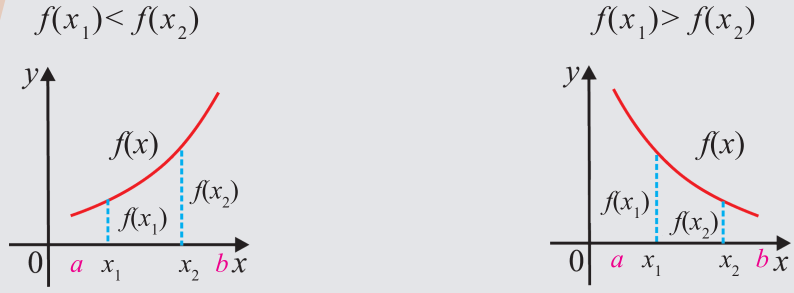
Funksiyalarning o'sishi va kamayishi

$f(x)$ funksiya $(a; b)$ oraliqda aniqlangan bo'lib, $x_1 < x_2$ shartni qanoatlantiruvchi barcha $x_1, x_2 \in (a; b)$ lar uchun:

- $f(x_1) < f(x_2)$ bo'lsa, $f(x)$ funksiya $(a; b)$ oraliqda o'suvchi;
- $f(x_1) > f(x_2)$ bo'lsa, $f(x)$ funksiya $(a; b)$ oraliqda kamayuvchi;
- $f(x_1) \geq f(x_2)$ bo'lsa, $f(x)$ funksiya $(a; b)$ oraliqda o'smaydigan;
- $f(x_1) \leq f(x_2)$ bo'lsa, $f(x)$ funksiya $(a; b)$ oraliqda kamaymaydigan funksiya deyiladi.

O‘suvchi, kamayuvchi, o‘smaydigan va kamaymaydigan funksiyalar umumiy nom bilan ***monoton funksiyalar*** deyiladi.

3-rasm



Funksiya ekstremum nuqtalari va ekstremumlari

- Agar:

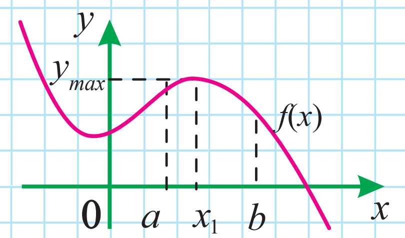
1) $f(x)$ funksiya x_1 nuqta tegishli bo‘lgan biror $(a;\ b)$ intervalda aniqlangan bo‘lib;

2) $(a;\ b)$ intervalning x_1 dan farqli barcha x nuqtalarida $f(x) < f(x_1)$ shart bajarilsa, u holda x_1 nuqta $f(x)$ ***funksiyaning maksimum nuqtasi*** deyiladi (4-*rasm*).

Agar $x_1 \in D(f)$ nuqta $f(x)$ funksiya uchun maksimum nuqta bo‘lsa, u holda $f(x)$ funksiyaning x_1 nuqtadagi $f(x_1)$ qiymati ***funksiyaning maksimumi*** deyiladi va $y_{\max}$ kabi belgilanadi. Demak,

$$y_{\max} = f(x_1).$$

4-rasm



$f(x)$ uchun $(a;\ b)$ intervalda
x_1 – funksiyaning maksimum nuqtasi;
$y_{max} = f(x_1)$ – funksiyaning maksimumi.

- Agar:

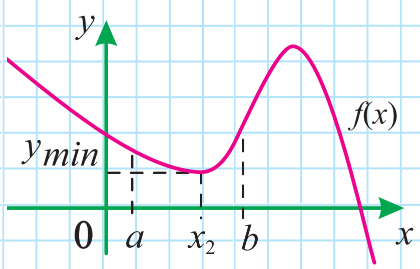
1) $f(x)$ funksiya x_2 tegishli bo‘lgan biror $(a;\ b)$ intervalda aniqlangan bo‘lib;

2) $(a;\ b)$ intervalning x_2 dan farqli barcha x nuqtalarida $f(x) > f(x_2)$ shart bajarilsa, u holda x_2 nuqta $f(x)$ ***funksiyaning minimum nuqtasi*** deyiladi (5-*rasm*).

Agar $x_2 \in X$ nuqta $f(x)$ funksiya uchun minimum nuqta bo‘lsa, u holda $f(x)$ funksiyaning x_2 nuqtadagi $f(x_2)$ qiymati $f(x)$ ***funksiyaning minimumi*** deyiladi va $y_{\min}$ kabi belgilanadi. Demak,

$$y_{\min} = f(x_2).$$

5-rasm



$f(x)$ uchun $(a;\ b)$ intervalda
x_2 – funksiyaning minimum nuqtasi;
$y_{\min} = f(x_2)$ – funksiyaning minimumi.

Funksiyaning maksimum va minimum nuqtalari ***ekstremum nuqtalari*** deyiladi.

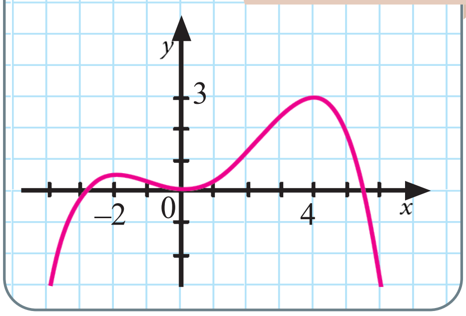
Funksiyaning ***ekstremum nuqtalardagi*** qiymatlari funksiya ***ekstremumlari*** deyiladi.

4-misol. $f(x)$ funksiyaning grafigi 6-rasmda keltirilgan. Funksiyaning o‘sish va kamayish oraliqlarini toping.

Yechish

$f(x)$ funksiyaning grafigidan funksiya $(-\infty;-2]$ va $[0;\ 4]$ oraliqlarda o‘sishini hamda $[-2;\ 0]$ va $[4;\infty)$ oraliqlarda kamayishini aniqlaymiz.

6-rasm



MISOLLAR

1. Berilgan funksiyalarning juft yoki toqligini tekshiring.

a) $f(x)=x^4$ b) $f(x)=x^3$

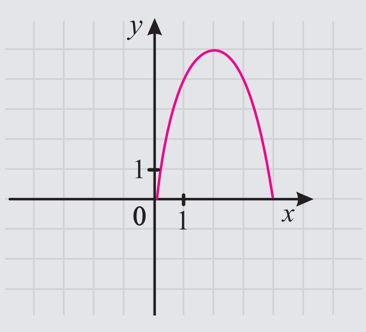
c) $f(x)=x^2+x$ d) $f(x)=x^4-4x^2$

e) $f(x)=x^3-x$ f) $f(x)=3x^3+2x^2+1$

g) $f(x)=1-\sqrt[3]{x}$ h) $f(x)=x+\dfrac{1}{x}$

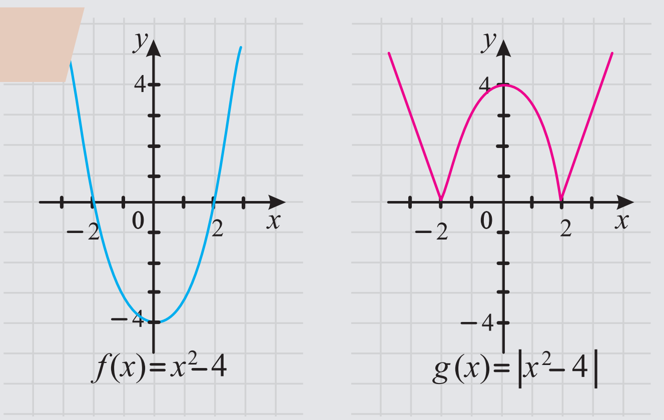
2. 7-rasmda $x\geq 0$ soha uchun funksiyaning grafigi berilgan. $x<0$ sohada grafikni shunday quringki:
1) juft funksiya;
2) toq funksiya grafigi hosil bo‘lsin.

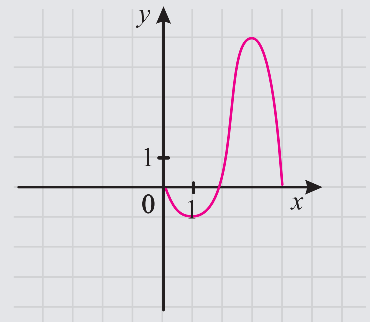
7-rasm



3. 8-rasmda $f(x)=x^2-4$ va $g(x)=|x^2-4|$ funksiya grafiklari berilgan. $g(x)$ funksiyaning grafigi $f(x)$ funksiyaning grafigidan qanday hosil qilinganini tushuntirib bering.

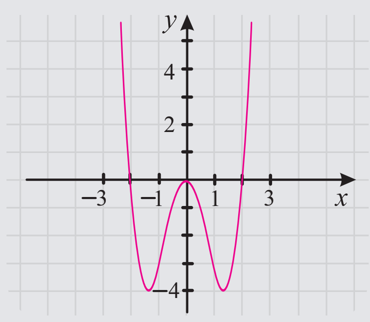
8-rasm





9-rasm

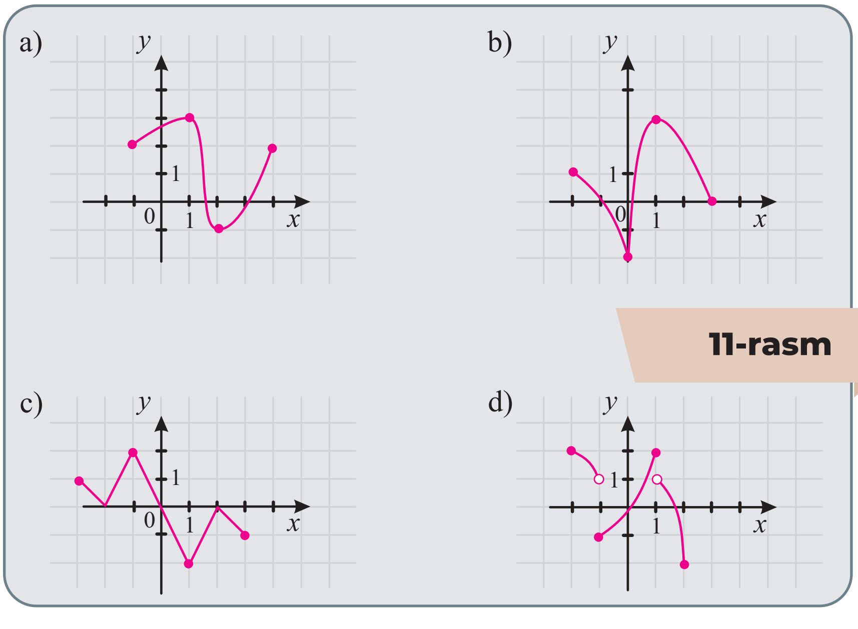
4. 9-rasmda $x \geq 0$ soha uchun funksiyaning grafigi berilgan. $x < 0$ sohada grafikni shunday quringki:
1) juft funksiya;
2) toq funksiya grafigi hosil bo'lsin.



10-rasm

5. $f(x) = x^4 - 4x^2$ funksiyaning grafigi berilgan (10-*rasm*). Undan foydalanib $g(x) = |x^4 - 4x^2|$ funksiyaning grafigini yasang.

6. 11-rasmda f funksiyaning grafigi berilgan. Bu grafikdan foydalanib quyidagilarni aniqlang:
1) f funksiyaning aniqlanish sohasini va qiymatlar to'plamini.
2) f funksiyaning o'sish va kamayish oraliqlarini.



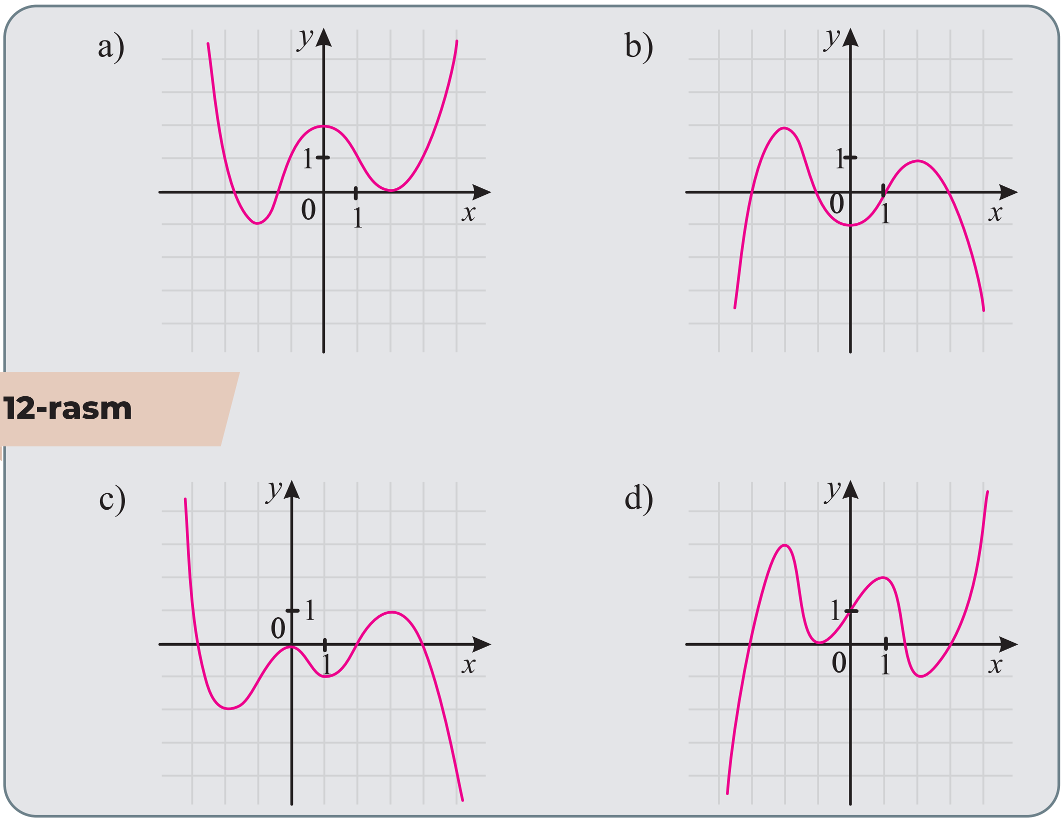
11-rasm

7. Berilgan funksiyalarning grafigini yasang, aniqlanish sohasini va qiymatlar to'plamini aniqlang, o'sish va kamayish oraliqlarini taxminiy toping.

a) $f(x)=x^2-5x$ b) $f(x)=x^3-4x$ c) $f(x)=x^4-16x^2$

8. f funksiyaning grafigi 12-rasmda berilgan. Bu grafikdan foydalanib quyidagilarni taxminiy aniqlang:

1) funksiyaning barcha ekstremum nuqtalarini va ekstremumlarini;
2) funksiyaning o'sish va kamayish oraliqlarini.



12-rasm

9. Quyidagi ma'lumotlar asosida funksiya grafigining eskizini yasang.

a) $(-\infty;3]$ kamayadi, $[3;+\infty)$ o'sadi;

b) $(-\infty;0]\cup[1;+\infty)$ kamayadi, $[0;1]$ o'zgarmaydi;

c) $(-\infty;-6]$ kamayadi, $[-6;0]$ o'sadi va $[0;+\infty)$ o'zgarmaydi;

d) $[-5;10]$ o'sadi, $[10;+\infty)$ o'zgarmaydi va $x=-5$ da eng kichik qiymatni qabul qiladi.

FUNKSIYA GRAFIGI USTIDA SODDA ALMASHTIRISHLAR

Funksiya grafigini siljitish

Berilgan $f(x)$ funksiyaning grafigini Oxy tekisligida siljitish mumkin. Funksiya grafigining quyida keltiriladigan siljitishlarini ko‘rib o‘tamiz.

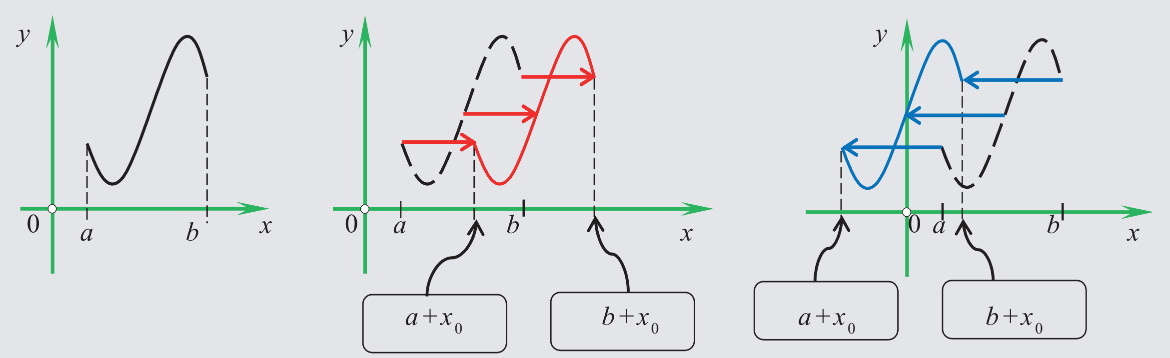
1. Funksiya grafigini Ox o‘qi bo‘yicha siljitish.
2. Funksiya grafigini Oy o‘qi bo‘yicha siljitish.
3. Funksiya grafigini biror vektor yo‘nalishida siljitish.

1. Funksiya grafigini Ox o‘qi bo‘yicha x_0 birlikka siljitish (1-*rasm*)

a) agar $x_0 > 0$ bo‘lsa, grafik Ox o‘qi yo‘nalishida x_0 birlik siljiydi.

b) agar $x_0 < 0$ bo‘lsa, grafik Ox o‘qi yo‘nalishiga qarshi $|x_0|$ birlik siljiydi.

1-rasm $y = f(x)$ funksiya grafigini Ox o‘qi bo‘yicha siljitish



a) berilgan $y = f(x)$ funksiya grafigi.

b) $x_0>0$ bo‘lganda $y = f(x-x_0)$ funksiya grafigi; $y = f(x)$ funksiya grafigi o‘ngga x_0 birlik siljigan.

c) $x_0<0$ bo‘lganda $y = f(x-x_0)$ funksiya grafigi; $y = f(x)$ funksiya grafigi chapga $|x_0|$ birlik siljigan.

1-misol. $f(x) = x^2$ funksiya grafigidan foydalanib quyidagi funksiyalar grafigini yasang.

a) $g(x)=(x+4)^2$ b) $h(x)=(x-2)^2$

Yechish. 2-rasmda ko‘rsatilgandek,

a) g funksiyaning grafigini yasash uchun f funksiya grafigini chapga 4 birlikka siljitamiz.

b) h funksiyaning grafigini yasash uchun f funksiya grafigini o‘ngga 2 birlikka siljitamiz.

2-rasm

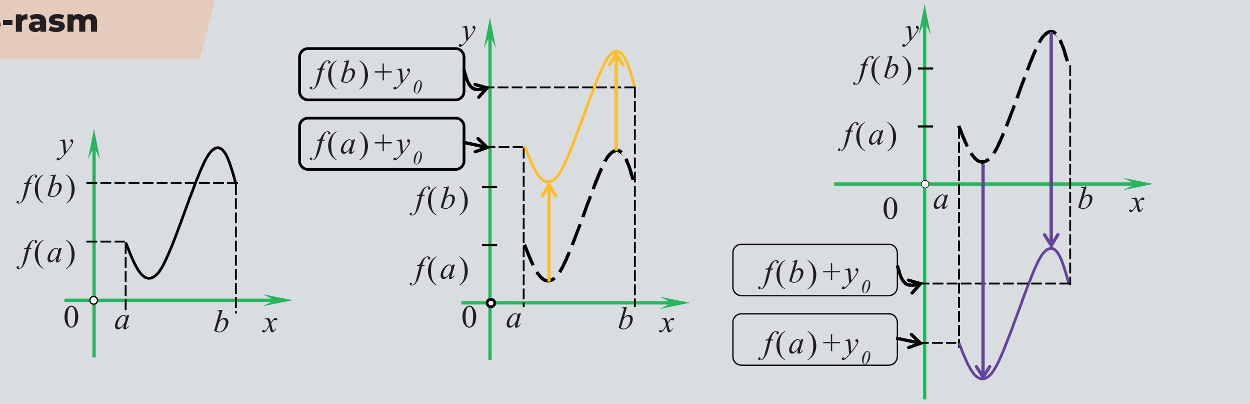
2. Funksiya grafigini *Oy* o‘qi bo‘yicha y_0 birlikka siljitish (3-*rasm*)

a) agar $y_0 > 0$ bo‘lsa, grafik Oy o‘qi yo‘nalishida y_0 birlik siljiydi;

b) agar $y_0 < 0$ bo‘lsa, grafik Oy o‘qi yo‘nalishiga qarshi $|y_0|$ birlik siljiydi (3-*rasm*).

3-rasm

$y = f(x)$ funksiya grafigini Oy o‘qi bo‘yicha siljitish.



a) Berilgan $y = f(x)$ funksiya grafigi.

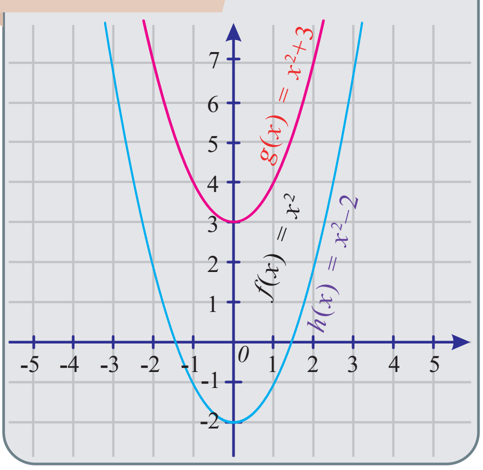
b) $y_0>0$ bo‘lganda $y = f(x)+y_0$ funksiya grafigi. $y = f(x)$ funksiya grafigi yuqoriga y_0 birlik siljigan.

c) $y_0<0$ bo‘lganda $y = f(x)+y_0$ funksiya grafigi. $y = f(x)$ funksiya grafigi pastga $|y_0|$ birlik siljigan.

2-misol. $f(x) = x^2$ funksiyadan foydalanib quyidagi funksiyalarning grafigini yasang.

a) $g(x) = x^2 + 3$ b) $h(x) = x^2 - 2$

4-rasm



Yechish

a) Quyidagiga e’tibor bering:

$$g(x) = x^2 + 3 = f(x) + 3$$

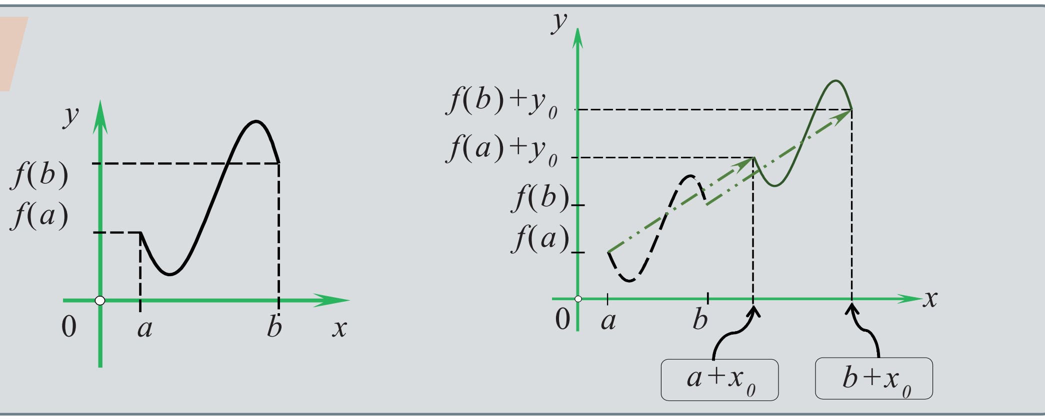
Demak, 4-rasmda ko‘rsatilgandek g funksiya grafigini chizish uchun f funksiyaning grafigini yuqoriga 3 birlikka siljitamiz (ko‘taramiz).

b) Xuddi shunday, h funksiyaning grafigini chizish uchun f funksiyaning grafigini pastga 2 birlikka siljitamiz (tushiramiz).

3. Funksiya grafigini ham *Ox*, ham *Oy* o‘qlari bo‘yicha siljitish (5-*rasm*)

$y = f(x - x_0) + y_0$ funksiya grafigini yasash uchun $y = f(x)$ funksiya grafigini Ox o‘qi bo‘yicha x_0 birlikka, Oy o‘qi bo‘yicha esa y_0 birlikka siljitish yetarli.

5-rasm

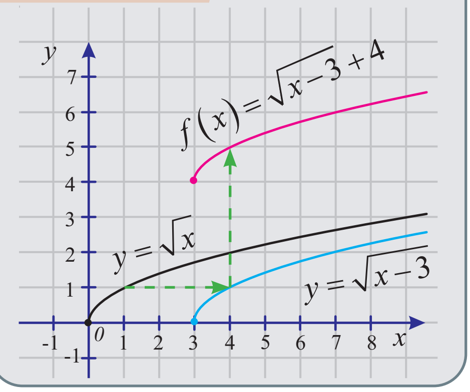


3-misol. $f(x)=\sqrt{x-3}+4$ funksiyaning grafigini yasang.

Yechish

Dastlab $y=\sqrt{x}$ funksiya grafigini yasaymiz. Hosil bo‘lgan funksiya grafigini 3 birlik o‘ngga siljitamiz va $y=\sqrt{x-3}$ funksiyaning grafigini hosil qilamiz. So‘ng bu grafikni 4 birlik yuqoriga siljitamiz va $f(x)=\sqrt{x-3}+4$ funksiya grafigini hosil qilamiz (6-*rasm*).

6-rasm



Funksiya grafiklarini siqish va cho‘zish

Berilgan $f(x)$ funksiyaning grafigini Oxy tekisligida deformatsiyalash (siqish yoki cho‘zish) mumkin.

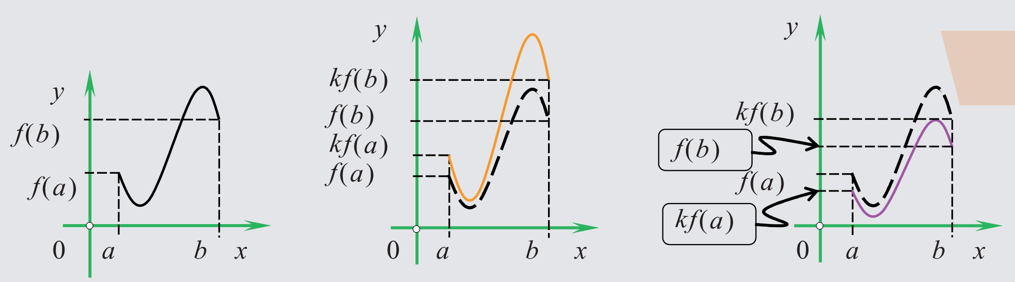
1-hol. Berilgan $y=f(x)$ funksiya grafigidan foydalanib $y=kf(x)$ funksiya grafigi quyidagicha hosil qilinadi (7-*rasm*):

a) agar $k>1$ bo‘lsa, grafik Ox o‘qidan Oy o‘qi bo‘yicha k baravar cho‘ziladi.

b) agar $0<k<1$ bo‘lsa, grafik Ox o‘qiga Oy o‘qi bo‘ylab $\frac{1}{k}$ baravar siqiladi.

c) agar $k<0$ bo‘lsa, u holda $y=kf(x)$ funksiya grafigi $y=|k|f(x)$ funksiya grafigining Ox o‘qqa nisbatan simmetrik aksi bo‘ladi.

$y=kf(x)$ funksiya grafigini hosil qilish



7-rasm

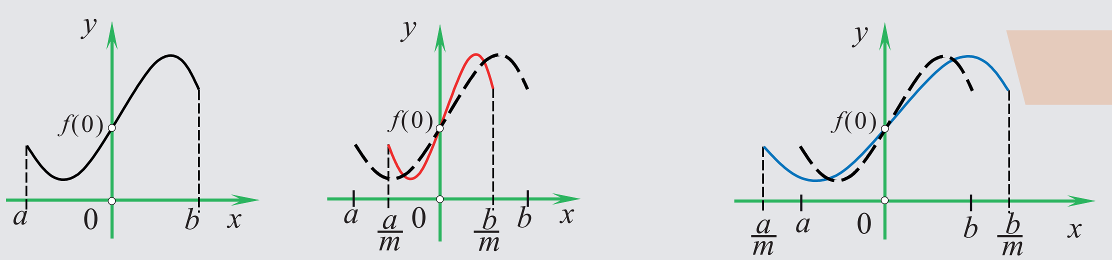
2-hol. Berilgan $y=f(x)$ funksiya grafigidan foydalanib $y=f(mx)$ funksiya grafigi quyidagicha hosil qilinadi (8-*rasm*):

a) agar $m>1$ bo‘lsa, grafik Oy o‘qiga Ox o‘qi bo‘ylab m baravar siqiladi;

b) agar $0<m<1$ bo‘lsa, grafik Oy o‘qidan Ox o‘qi bo‘ylab $\frac{1}{m}$ baravar cho‘ziladi.

c) agar $m<0$ bo‘lsa, u holda $y=f(mx)$ funksiyaning grafigi $y=f(|m|x)$ funksiya grafigining Oy o‘qqa nisbatan simmetrik aksi bo‘ladi.

$y=f(mx)$ funksiya grafigini hosil qilish



8-rasm

4-misol. Quyidagi funksiyaning grafigini yasang.

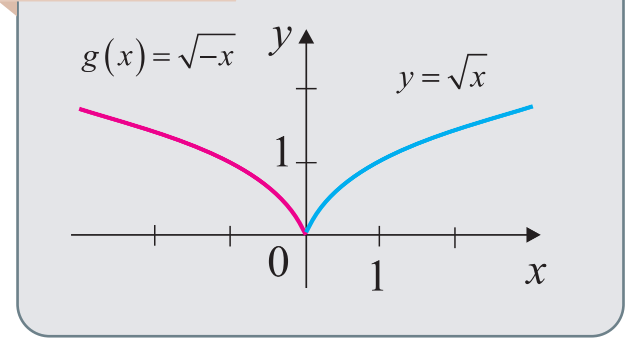
$$g(x)=\sqrt{-x}$$

Yechish

9-rasmda $y=\sqrt{x}$ funksiyaning grafigini chizamiz. Bu grafikni y o'qiga nisbatan simmetrik akslantirish orqali $g(x)=\sqrt{-x}$ funksiyaning grafigini hosil qilamiz.

E'tibor qiling: $g(x)=\sqrt{-x}$ funksiyaning aniqlanish sohasi: $x\leq 0$ dan iborat.

9-rasm



5-misol. 10-rasmda $f(x)=x^2$ funksiyaning grafigidan foydalanib quyidagi funksiyalarning grafigini yasang.

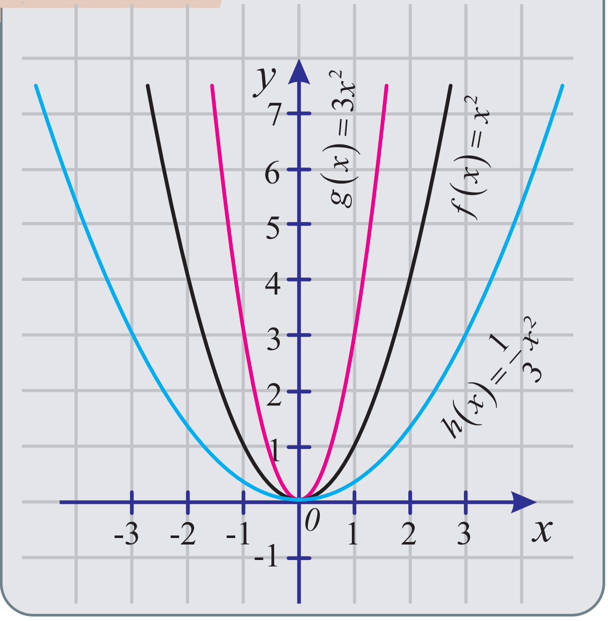
a) $g(x)=3x^2$ b) $h(x)=\frac{1}{3}x^2$

Yechish

a) g funksiyaning grafigi f funksiyaning har bir nuqtasining y koordinatasini 3 ga ko'paytirishdan hosil bo'ladi. Ya'ni g funksiyaning grafigini hosil qilish uchun f funksiyaning grafigini vertikal 3 baravar cho'zish kerak.

b) h funksiyaning grafigi f funksiyaning har bir nuqtasining y koordinatasini $\frac{1}{3}$ ga ko'paytirishdan hosil bo'ladi. Ya'ni h funksiyaning grafigini hosil qilish uchun f funksiyaning grafigini vertikal yo'nalishda x o'qqa 3 baravar siqish kerak.

10-rasm

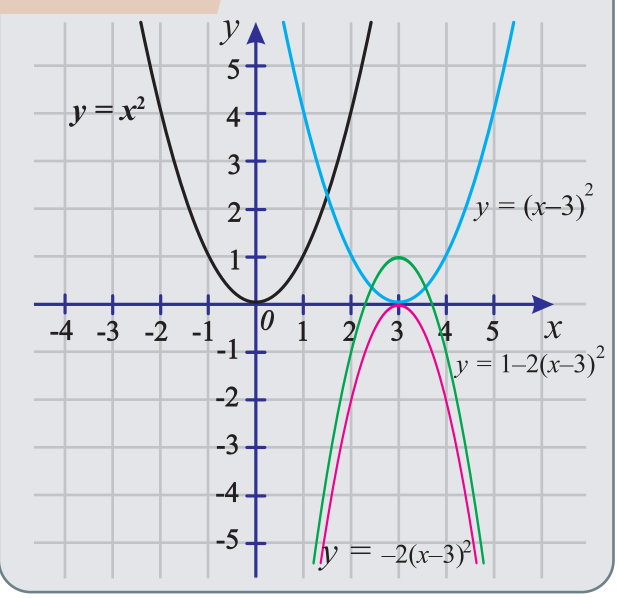


6-misol. $f(x)=1-2(x-3)^2$ funksiyaning grafigini yasang.

Yechish

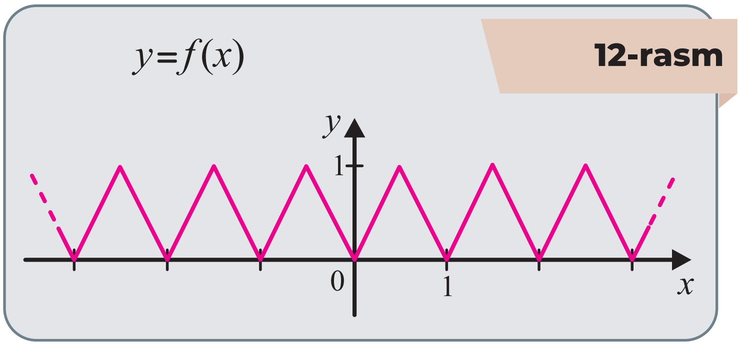
Dastlab $y=x^2$ funksiyaning grafigini o'ngga 3 birlikka gorizontal siljitamiz va $y=(x-3)^2$ funksiyaning grafigini hosil qilamiz. Keyin bu grafikni Ox o'qiga nisbatan simmetrik akslantiramiz. Oy o'qi bo'yicha 2 baravar cho'zishni bajaramiz va $y=-2(x-3)^2$ funksiyaning grafigini hosil qilamiz. Nihoyat bu grafikni yuqoriga 1 birlikka siljitamiz va $f(x)=1-2(x-3)^2$ funksiyaning grafigini hosil qilamiz. (11-*rasm*).

11-rasm



7-misol. 12-rasmda $y=f(x)$ funksiyaning grafigi berilgan. Bu grafikdan foydalanib quyidagi funksiyalarning grafigini yasang.

a) $y=f(2x)$; b) $y=f\left(\frac{1}{2}x\right)$.

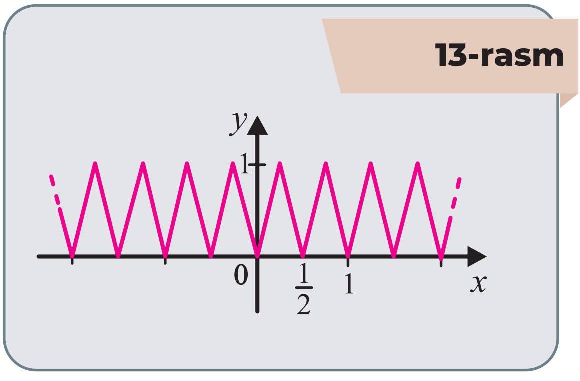


12-rasm

Yechish

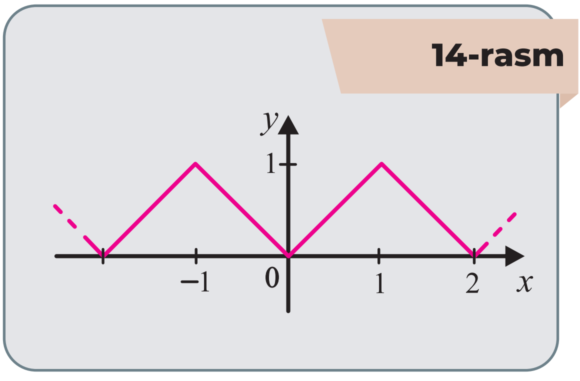
a) $y=f(2x)$ funksiya grafigini yasash uchun $f(x)$ funksiya grafigini *Oy* o'qiga *Ox* o'qi bo'yicha 2 baravar siqamiz (13-*rasm*).

b) $y=f\left(\frac{1}{2}x\right)$ funksiya grafigini yasash uchun $f(x)$ funksiya grafigini *Oy* o'qidan *Ox* o'qi bo'yicha 2 baravar cho'zamiz (14-*rasm*).



13-rasm

$y=f(2x)$



14-rasm

$y=f\left(\frac{1}{2}x\right)$

MISOLLAR

1. $f(x)$ funksiya grafigi berilgan bo'lsa, quyidagi funksiyalarning grafigi qanday yasalishini tushuntiring.

a) $y=f(x)-1$ b) $y=f(x-2)$ c) $y=f\left(\frac{1}{4}x\right)$

d) $y=f(x)+4$ e) $y=f(-x)$ f) $y=3f(x)$

g) $y=-f(x)$ h) $y=\frac{1}{3}f(x)$ i) $y=f(x-5)+2$

j) $y=f(x+1)-1$ k) $y=4f(x+1)+3$ l) $y=f(4x)$

m) $y=-f(x)+5$ n) $y=3f(x)-5$ o) $y=1-f(-x)$

2. g funksiyaning grafigi f funksiyaning grafigidan qanday almashtirishlar yordamida hosil qilinganini tushuntiring.

a) $f(x)=x^2,\ g(x)=(x+2)^2$
b) $f(x)=x^2,\ g(x)=x^2+2$
c) $f(x)=x^3,\ g(x)=(x-4)^3$
d) $f(x)=x^3,\ g(x)=x^3-4$
e) $f(x)=|x|,\ g(x)=|x+2|-2$
f) $f(x)=|x|,\ g(x)=|x-2|+2$
g) $f(x)=\sqrt{x},\ g(x)=-\sqrt{x}+1$
h) $f(x)=\sqrt{x},\ g(x)=\sqrt{-x}+1$

3. $y=x^2$ funksiyaning grafigidan foydalanib quyidagi funksiyalarning grafigini chizing.

a) $g(x)=x^2+1$
b) $g(x)=(x-1)^2$
c) $g(x)=-x^2$
d) $g(x)=(x-1)^2+3$

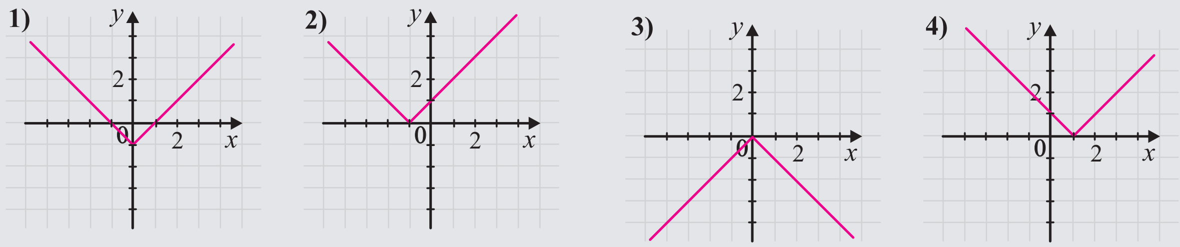
4. $y=\sqrt{x}$ funksiyaning grafigidan foydalanib quyidagi funksiyalarning grafigini chizing.

a) $g(x)=\sqrt{x-2}$
b) $g(x)=\sqrt{x}+1$
c) $g(x)=\sqrt{x+2}+2$
d) $g(x)=-\sqrt{x}+1$

5. Berilgan funksiyalarga 15-rasmda berilgan grafiklardan mosini toping.

a) $y=|x+1|$
b) $y=|x|-1$
c) $y=|x-1|$
d) $y=-|x|$

15-rasm



6. Quyidagi funksiyalarning grafiklarini standart funksiyaning grafigi ustida mos almashtirishlarni bajarib chizing.

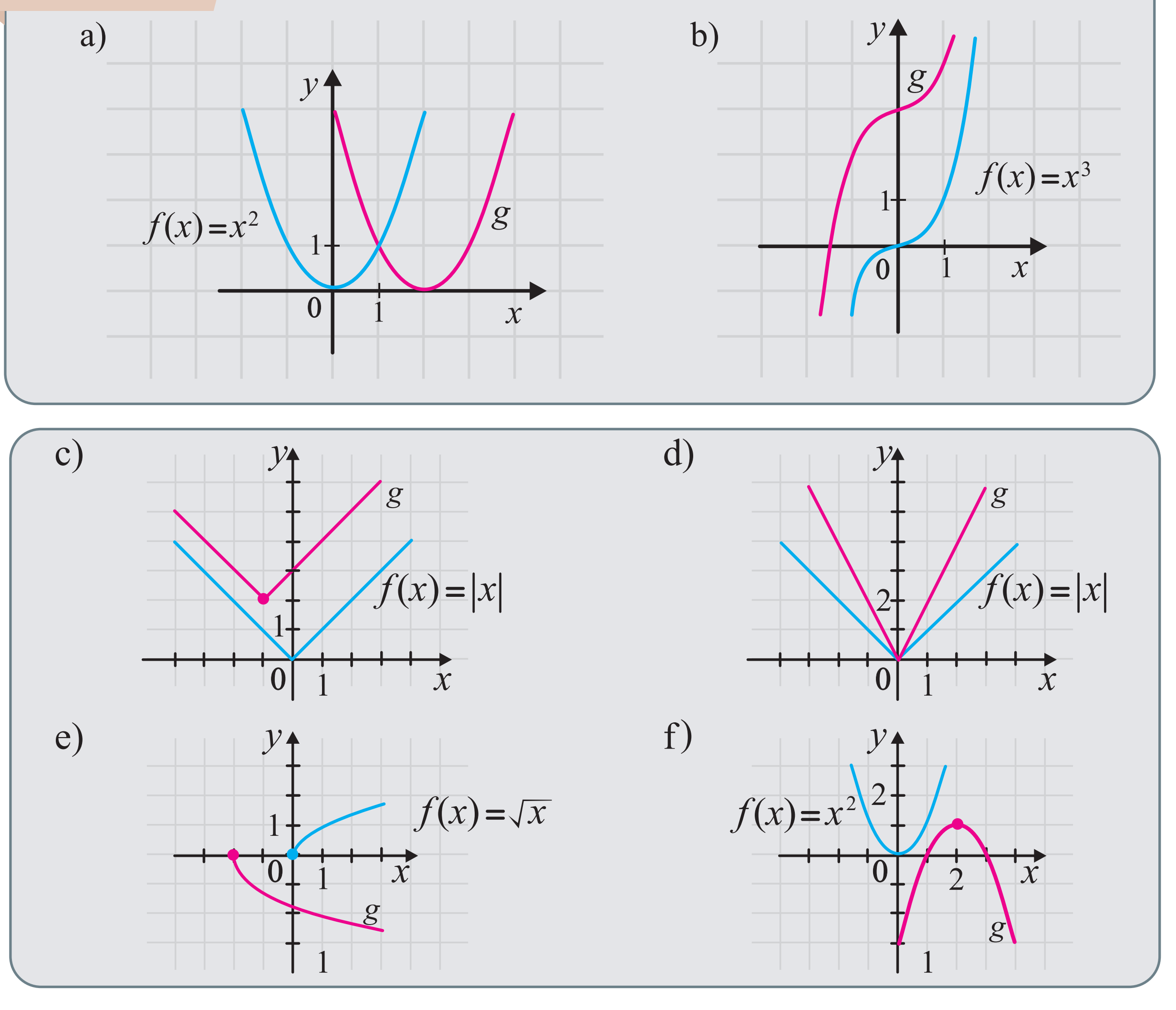
a) $f(x)=x^2+3$
b) $f(x)=\sqrt{x}+1$
c) $f(x)=|x|-1$
d) $f(x)=\sqrt{x}+1$
e) $f(x)=(x-5)^2$
f) $f(x)=(x+1)^2$
g) $f(x)=|x+2|$
h) $f(x)=\sqrt{x-4}$
i) $f(x)=-x^3$
j) $f(x)=-|x|$
k) $y=\sqrt[4]{-x}$
l) $y=\sqrt[3]{-x}$
m) $y=\dfrac{1}{4}x^2$
n) $y=-5\sqrt{x}$
o) $y=3|x|$
p) $y=\dfrac{1}{2}|x|$
q) $y=(x-3)^2+5$
r) $y=\sqrt{x+4}-3$
s) $y=3-\dfrac{1}{2}(x-1)^2$
t) $y=2-\sqrt{x+1}$
u) $y=|x+2|+2$
v) $y=2-|x|$
w) $y=\dfrac{1}{2}\sqrt{x+4}-3$
x) $y=3-2(x-1)^2$.

7. Berilgan f funksiyaning grafigiga ko'rsatilgan almashtirishlar qo'llangan. Yakuniy funksiyaning formulasini yozing.

a) $f(x)=x^2$, 3 birlik pastga siljiting.
b) $f(x)=x^3$, 5 birlik yuqoriga siljiting.
c) $f(x)=\sqrt{x}$, 2 birlik chapga siljiting.
d) $f(x)=\sqrt[3]{x}$, 1 birlik o'ngga siljiting.
e) $f(x)=|x|$, 2 birlik chapga va 5 birlik pastga siljiting.
f) $f(x)=|x|$, x o'qiga nisbatan akslantirib, 4 birlik o'ngga va 3 birlik yuqoriga siljiting.
g) $f(x)=\sqrt[4]{x}$, y o'qiga nisbatan simmetrik akslantiring va 1 birlik yuqoriga siljiting.
h) $f(x)=x^2$, 2 birlik chapga siljiting va x o'qiga nisbatan simmetrik akslantiring.
i) $f(x)=x^2$, 2 baravar vertikal cho'zib, 2 birlik pastga va 3 birlik o'ngga siljiting.
j) $f(x)=|x|$, $\frac{1}{2}$ baravar vertikal yo'nalishda siqishni bajarib, 1 birlik chapga va 3 birlik yuqoriga siljiting.

8. f va g funksiyalarning grafigi berilgan (16-*rasm*). f funksiyadan foydalanib g funksiyaning formulasini toping.

16-rasm



9. $y=f(x)$ funksiya berilgan, 17-rasmda quyidagilarga mos grafikni toping.

a) $y=f(x-4)$

b) $y=f(x)+3$

c) $y=2f(x+6)$

d) $y=-f(2x)$

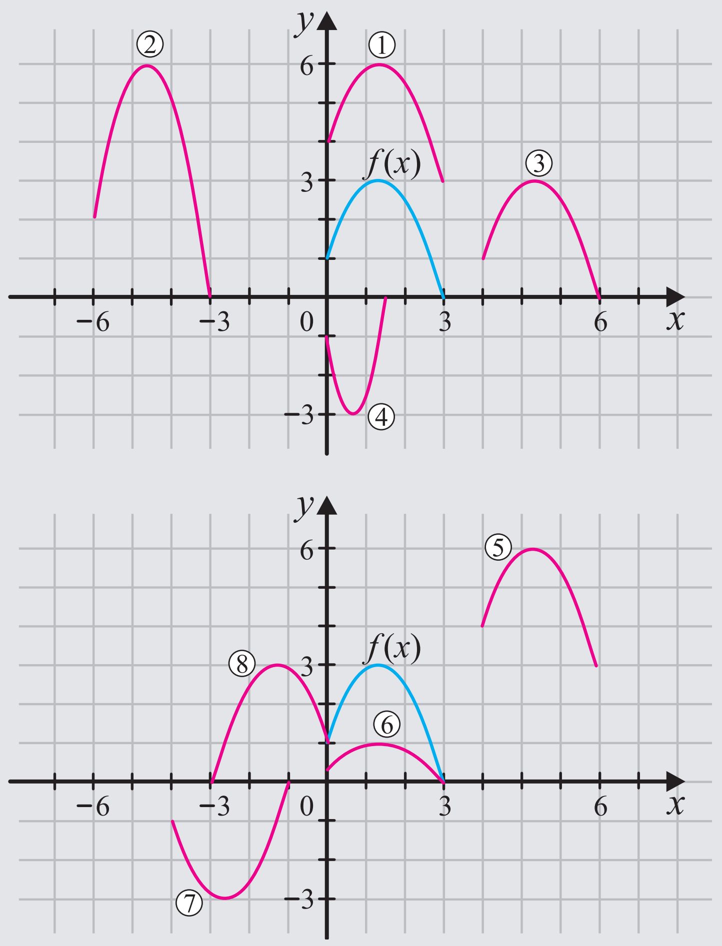
e) $y=\frac{1}{3}f(x)$

f) $y=-f(x+4)$

g) $y=f(x-4)+3$

h) $y=f(-x)$

17-rasm



CHIZIQLI VA KVADRATIK MODELLASHTIRISHLAR

Matematik modellashtirish kundalik hayotimizdagi turli jarayonlarni o‘rganishning asosiy analitik vositasi hisoblanadi.

Quyidagi masalalarni ko‘rib chiqamiz.

1-masala. Porshenli nasos eng ko‘pi bilan qanday chuqurlikdan suv chiqara olishini toping (1-*rasm*).

Yechish

Ma'lumki, porshenli nasos trubasidagi suv ustunining bosimi

$$p = \rho g h$$

formula bilan hisoblanadi.

Bu yerda $\rho = 1000\ \text{kg/m}^3$ – suv zichligi, $g = 10\ \text{m/s}^2$ – erkin tushish tezlanishi, h – suv ustunining balandligi.

Nasos yer sathida joylashgani uchun suv ustuni balandligi *suv chuqurligi* deb yuritiladi. Demak, suv chuqurligini

$$h = \frac{p}{\rho g}$$

tenglikdan topish mumkin.

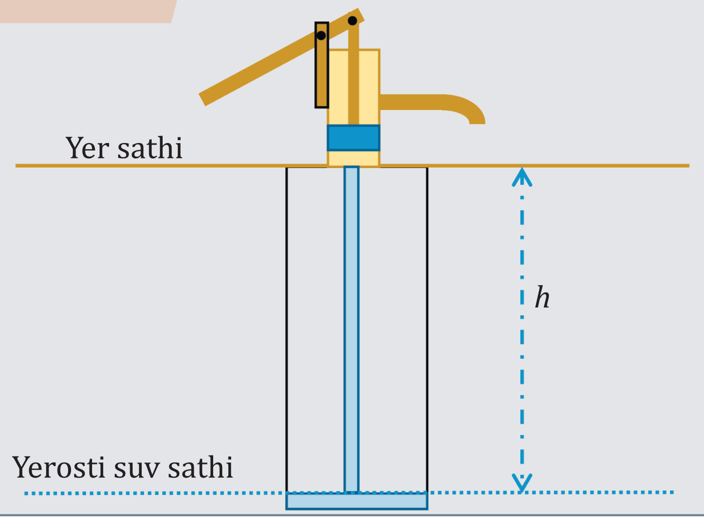
1643-yilda italyan fizigi Evangelista Torrichelli tajribalarida suv ustuni bosimi atmosfera bosimi p_0 = 100 000 Pa dan oshib ketmasligi isbotlangan, ya'ni $p \le p_0$. Shuning uchun porshenli nasosda suv chuqurligi

$$h = \frac{p}{\rho g} \le \frac{p_0}{\rho g} = \frac{100000}{1000 \cdot 10} = 10\ \text{m}$$

dan oshib keta olmas ekan.

Javob: Porshenli nasos eng ko‘pi bilan 10 m chuqurlikdan suv chiqara oladi.

1-rasm



Bu modeldagi o‘zgaruvchilar birinchi darajali bo‘lib, o‘zaro chiziqli amallar (qo‘shish va songa ko‘paytirish) orqali bog‘langan. Shuning uchun bu tipdagi matematik modellar ***chiziqli modellar*** deyiladi. Qo‘yilgan masalani chiziqli model shakliga olib kelish jarayoni ***chiziqli modellashtirish*** deyiladi.

2-masala. O‘quvchi *Oxy* koordinatalar tekisligini shunday tanladiki, bunda o‘z uyini koordinata boshi $O(0;\ 0)$ deb oldi. Keyin o‘zi o‘qiydigan maktab $C(4;\ 3)$ nuqtada joylashganini aniqladi. Yo‘lning uyi va maktab orasidan o‘tadigan to‘g‘ri chiziqli qismi *Ox* o‘qini (6; 0) nuqtada, *Oy* o‘qini (0; 4) nuqtada kesib o‘tishini hisoblab chiqdi.

Maktabga uyali aloqa kompaniyasining antennasi o‘rnatilganligi ma’lum. O‘quvchi yo‘lda harakatlanayotgan avtomobildagi yo‘lovchining uyali aloqa vositasi antennadan tarqalayotgan to‘lqinni eng yaxshi tutadigan nuqtani topishga qiziqib qoldi.

Topshiriq. Siz bu masalani qanday yechgan bo‘lar edingiz?

Yechish. Ravshanki, yo‘lning maktabga eng yaqin nuqtasida uyali aloqa vositasi to‘lqinni eng yaxshi tutadi. Bu masalani yechishda yo‘lni tavsiflovchi (*AB*) to‘g‘ri chiziq tenglamasini tuzish va uning maktabga eng yaqin nuqtasining koordinatalarini topish kerak. Buning uchun avvalo bayon etilganlar asosida vaziyatning chizmasi chiziladi (2-*rasmga qarang*).

Keyin $A(6;\ 0)$ va $B(0;\ 4)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi tuziladi. Buning uchun to‘g‘ri chiziqning

$$y = kx+b$$

tenglamasiga $A(6;\ 0)$ va $B(0;\ 4)$ nuqtalarning koordinatalarini qo‘yib, ushbu

$$0 = k\cdot 6+b$$
$$4 = k\cdot 0+b$$

tengliklar hosil qilinadi. Ulardan

$$b = 4,\ \ k = -\frac{2}{3}$$

koeffitsiyentlar topiladi. Demak, (*AB*) to‘g‘ri chiziq tenglamasi

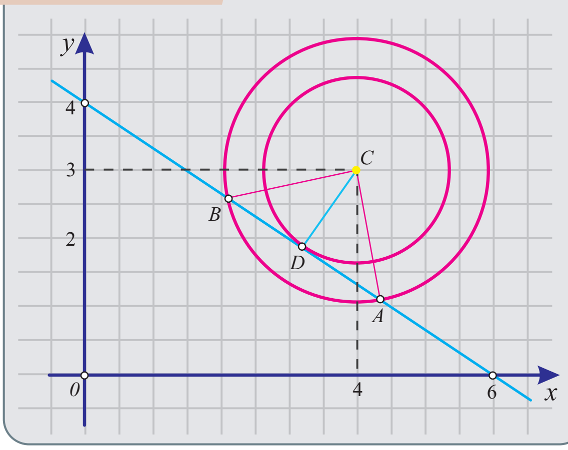
$$y = -\frac{2}{3}x+4$$

bo‘ladi.

Masalaning yechimi (*AB*) to‘g‘ri chiziqning $C(4;\ 3)$ nuqtaga eng yaqin $D(x;\ y)$ nuqtasini topishdan iborat. Bu vaziyatning matematik modeli quyidagicha yoziladi:

$$F = \sqrt{(x-4)^2+(y-3)^2} \to min,$$
$$y = -\frac{2}{3}x+4$$

2-rasm



Bu modeldagi o‘zgaruvchilar birinchi va ikkinchi darajali bo‘lgani uchun bu tipdagi matematik modellar ***kvadratik modellar*** deyiladi. Qo‘yilgan masalani kvadrat model shakliga olib kelish jarayoni ***kvadratik modellashtirish*** deyiladi.

MISOLLAR

1. Har bir berilgan topshiriqning chiziqli modelini yozing:

a) Siz velosipedni 10 000 so'm boshlang'ich to'lov va soatiga 5000 so'm tarif bo'yicha ijaraga oldingiz.

b) Avtomobillarni ta'mirlash ustaxonasi 50 000 so'm bazaviy to'lov hamda soatiga 15 000 so'mdan to'lov belgiladi.

c) Shamning uzunligi 30 cm va u soatiga 1,4 cm tezlikda yonadi.

d) Dasturlash bo'yicha mutaxassis maslahat uchun alohida $75 va undan so'ng soatiga $35 oladi.

e) Hozirgi harorat 25 °C va u kechasi har soatda 2 °C ga tushishi kutilmoqda.

f) Qishloq aholisi 6791 kishini tashkil etadi va yiliga 7 taga kamayib bormoqda.

2. Berilgan jadvaldagi funksiya chiziqli yoki kvadratik ekanini aniqlang.

x	0	1	3	4	6
y	5	10	20	25	35

3. To'p tepaga va pastga sakraganda uning erishadigan balandligi doimiy ravishda kamayadi. Quyidagi jadvalda vaqt bo'yicha sakrash balandligi ko'rsatilgan.

a) Eng mos keladigan kvadrat funksiyani toping.

b) To'pning maksimal balandligini toping.

c) 2,5 sekundda to'pning qancha balandlikda bo'lganini taxmin qiling.

t (s)	2	2,2	2,4	2,6	3
h (dyuym)	2	16	26	33	42

4. Agar tosh 70 metrli binoning tepasidan otilgan bo'lsa, toshning vaqtga bog'liq balandligi $h(t) = -5t^2 - 20t + 70$ kvadrat funksiya bilan berilgan, bu yerda t sekundda, balandligi esa metrda. Necha sekunddan keyin tosh yerga tegadi?

5. Malika xonasini tozalashga Umidadan ikki baravar ko'p vaqt sarflaydi. Aziza xonasini tozalashi uchun Umidadan 10 minut ko'proq vaqt ketkazadi. Ular xonalarini tozalash uchun jami 90 minut sarflaydi. Malika xonasini tozalashi uchun qancha vaqt sarflaydi?

6. Dilshod dengizga durni olish uchun sho'ng'idi. Uning t sekunddan keyingi sho'ng'ish chuqurligi $h(t) = -4t^2 + 4t + 3$ metr bo'ldi, $t \geq 0$.

a) durlar qanday chuqurlikda joylashgan?

b) Dilshod durni olish uchun qancha vaqt sarflaydi?

c) Dilshod qanday balandlikdan suvga sho'ng'idi?

7. Jasmina ko'ylak tikish uchun buyurtma oldi. U bir kunda x ta ko'ylak tiksa, $P(x) = -x^2 + 20x$ ming so'm miqdorida daromad oladi.

a) Eng katta daromad olish uchun u qancha ko'ylak tikishi kerak?

b) Eng katta daromad necha so'mga teng?

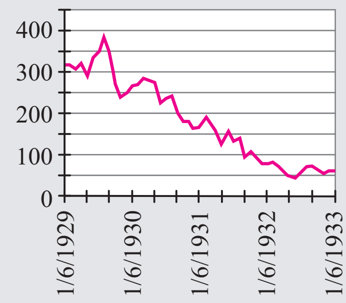
8. 2005-yilda Zarafshon shahri aholisi 55 000 ga yaqin edi. O'sha paytda aholi soni yiliga 2000 ga yaqin sur'atlarda o'sib borardi. Har qanday yil uchun Zarafshon aholisini topish zarur. Buning uchun uning chiziqli modelini tuzing. 2010-yili Zarafshon aholisi qancha bo'lgan? Zarafshon aholisi soni 2025-yili qancha bo'lishini hisoblang.

LOYIHA ISHI

Har bir grafik hikoya qiladi

Agar rasm ming so‘zga arzigulik bo‘lsa, unda grafik hech bo‘lmaganda bir necha qator jumlalariga arziydi. Darhaqiqat, grafik ba’zan hikoyani ko‘p so‘zlarga qaraganda tezroq va samaraliroq aytib berishi mumkin. 1929-yildagi fond bozori qulashining halokatli ta’siri Dou Jons indeksining (DJIA) grafigidan (1-*rasm*) darhol ko‘rinadi. O‘sha paytdagi gazetalarda bunday grafiklar halokatning naqadar kattaligini yetkazishning samarali usuli sifatida chop etilgan.

1-rasm



2-rasmdagi xabarni yetkazish uchun hech qanday so‘z kerak emas. Grafik oddiy bir voqeani aytib beradi: nimadir pastga tushdi – ehtimol, savdo, foyda yoki mahsuldorlik, mas’ul shaxs esa juda xavotirda.

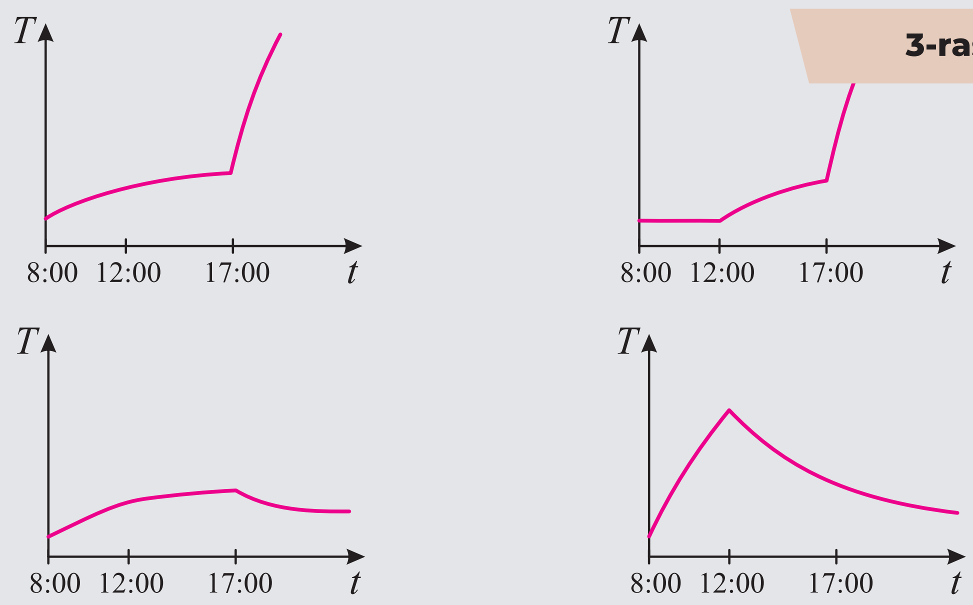
2-rasm

Ushbu loyiha tadqiqotida biz grafiklar aytib beradigan hikoyalarni o‘qiymiz va hikoya qiluvchi grafiklarni yaratamiz.

Hikoyani grafikdan o‘qish

1. Quyida haroratning vaqtga nisbatan to‘rtta grafigi (soat 8:00 dan boshlab) ko‘rsatilgan bo‘lib, ulardan keyin uchta hikoya berilgan. (3-*rasm*).

3-rasm



a) hikoyalarning har birini grafiklarning biri bilan moslang.

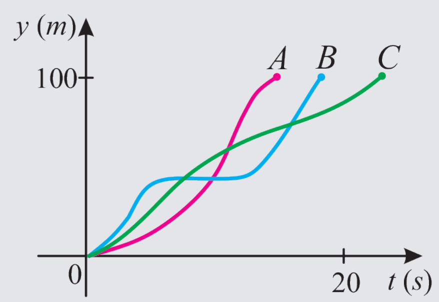
b) hech qanday hikoyaga mos kelmaydigan grafik uchun xuddi shunday hikoya yozing.

1-hikoya	Tushda muzlatkichdan go'shtni olib, eritish uchun peshtaxtaga qo'ydim va ishga ketdim. Ishdan uyga qaytganimdan keyin go'shtni pechda pishirdim.
2-hikoya	Ertalab muzlatkichdan go'shtni olib, eritish uchun peshtaxtaga qo'ydim va ishga ketdim. Ishdan uyga qaytganimdan keyin go'shtni pechda pishirdim.
3-hikoya	Ertalab muzlatkichdan go'shtni olib, eritish uchun peshtaxtaga qo'ydim va ishga ketdim. Men buni unutib, ishdan uyga qaytayotganimda kafeda tanovul qildim. Uyga kelganimdan so'ng esa go'shtni muzlatkichga qayta qo'ydim.

2. 100 metrga to'siqlar osha yugurishda uchta yuguruvchi qatnashdi. Grafikda yugurish masofasi har bir yuguruvchi uchun vaqt funksiyasi sifatida ko'rsatilgan (4-*rasm*). Grafik sizga ushbu poyga haqida nima deyishini tasvirlab bering. Poygada kim g'olib chiqdi? Har bir yuguruvchi poygani tugatdimi? Sizningcha, B sportchi bilan nima sodir bo'lgan?

3. Quyidagi chizmaga mos keladigan (har qanday vaziyatni o'z ichiga olgan) hikoya tuzing. (5-*rasm*).

4-rasm



5-rasm